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Calibrating partial factors for Danish railway embankments using probabilistic analyses

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ABSTRACT

High costs are connected with upgrading railway embankments throughout Denmark using the partial factors for geotechnical design calibrated for general application. One way to reduce the costs is reliability-based calibration of the partial factors to a reasonable safety level taking into account the specific design situations and uncertainties relevant to railway embankments. A reliability-based design has been investigated, resulting in an optimal partial factor for the considered subsoil. With a stochastic soil model to simulate the undrained shear strength of soft soil deposits, the partial factor is calibrated using asymptotic sampling for the reliability assessment. The calibration shows that the partial factor can be reduced significantly compared to the value specified in the Danish National Annex to DS/EN 1997-1 (2007), Eurocode 7.

1. Introduction

In Denmark, many existing railways are upgraded to higher train velocities and more tracks, leading to increasing demands concerning the structural safety. With the current railway codes, a railway embankment which is upgraded with an extra track must fullfil consequence class CC3 in Eurocode 7 (Banedanmark, 2010). These requirements are expensive, making embankments adequately safe but not excessively safe.

The partial factor in the Danish National Annex to DS/EN 1997-1 (2007) has been calibrated for application to a wide range of structures. In Denmark, the partial factor for the undrained shear strength is rather high, $\gamma_{c_u} = 1.8K_{FI}$, compared to the general factor proposed in Eurocode 7, $\gamma_{c_u} = 1.4K_{FI}$, where the factor K_{FI} accounts for the severity of the potential failure. This leads the embankment design to become very expensive.

Using the partial factor method (DS/EN 1990, 2007) for specific design of railway embankments, special considerations can be taken into railway design. The partial factor method ensures a certain safety level for the structure, but this level is unknown to the designer since he/she has no influence on the selected partial factors proposed in the Danish National Annex to Eurocode 7. Specifying the partial factor makes it easy for the designer to introduce safety in the structure, but if the partial factors are not chosen with care, the structure could end up being too safe, and thus very expensive.

Rather than defining a set of partial factors to be used in the design process, a required level of safety could be defined. This will make the design process more complex, but also more flexible allowing the designer to choose a set of partial factors that will ensure the required safety level for the structure. This could be accomplished using probabilistic design of the embankment (Ching et al., 2011). Here all parameters are modelled by stochastic variables with a distribution function, a mean value and a standard deviation (Koudelka, 2011).

The concept of probabilistic design is presented in the paper. Probabilistic modelling of soil and loads are described, and a sampling technique is provided to minimize calculation time. In the analyses conducted by Lodahl et al. (2012), it was found that the current general demands in the Danish National Annex to Eurocode 7 are too conservative for a railway embankment in cohesive fill situated on soft soils.

The analyses presented are based on the utilization of circular slip surface, and two embankments designed with the optimized partial factor are investigated. Furthermore, modelling with the purpose of estimating the most probable slip surfaces for each of the embankments is carried out.

The analyses are based on a typical situation when upgrading existing railway tracks in Denmark, namely embankments built

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with cohesive clay fill situated on soft soils, are often peat or gyttja. The embankments are often old, with consolidation effects resulting in harder soils under the embankment than that beside it. In the presented analyses, embankments of different heights, situated on different thicknesses of soft soils, are analysed.

2. Probabilistic modelling

In the probabilistic design method, a *limit state equation* is formulated. This equation is in general formulated as the load bearing capacity or resistance, R, minus the loading, L. Each of these can be modelled as dependent on a number of stochastic variables:

$$g(\boldsymbol{x}) = R(\boldsymbol{x}_R) - L(\boldsymbol{x}_L) \tag{1}$$

where $\boldsymbol{x}_R$ and $\boldsymbol{x}_L$ contain the uncertain parameters applied in model R and L, respectively; and $\boldsymbol{x} = (\boldsymbol{x}_R, \boldsymbol{x}_L)$. When the outcomes of the limit state equation are positive, i.e. $g(\boldsymbol{x}) > 0$, the structure is 'safe', whereas a negative limit state equation means failure. The probability of failure, P_f, is calculated as the probability of the limit state equation being negative, and is linked to the *reliability index*, β, by

$$\beta = \Phi^{-1}\left(-P_f\right), \quad P_f = \int_{g(x)>0} f_x(x)\mathrm{d}x \tag{2}$$

where $P_f = P(g(\boldsymbol{x}) \leq 0)$. The theoretical background for structural reliability can be found in various literature, e.g. Madsen et al. (1986) and Ditlevsen and Madsen (1996).

The integral in Eq. (2) may be solved by first order reliability method (FORM) or simulation, of which the latter will be discussed in this paper. The use of failure simulation techniques is generally recommended when the limit state equation is discontinuous. Since the determination of the resistance in this case is an iterative process, more computational time is needed to evaluate the limit state equation. Moreover, the probabilities to be estimated are very low (of the order of 10^{-5}–10^{-7}) since high-reliability indices are required. This suggests that advanced simulation techniques have to be applied in the reliability assessment. That is why the asymptotic sampling technique described by Bucher (2009) is presented in this paper.

2.1. Limit state equation

The factor of safety, FS, for an embankment is defined using the following equation:

$$FS = \frac{M_{\text{stab}} + z_{\text{opt}}}{M_{\text{driv}}}; \quad FS \geq 1 \tag{3}$$

where M_{stab} (kN m/m) and M_{driv} (kN m/m) are the stabilising and driving moments around the point of rotation, respectively. The centre of rotation is selected so that the factor of safety, FS, is minimized. An optimization procedure is performed to determine the location of the rotation centre in order to find the minimum FS.

Since both the driving moment and the stabilising moment can be calculated in different ways, the factor of safety, FS, is not directly a measure of the reliability of the structure. FS is determined using characteristic values of the load and resistance parameters combined with the given set of partial factors.

If the minimum FS is less than 1, the embankment is unstable and actions increasing the stabilising moment should be considered. That is why z_{opt} is introduced as a *design parameter*, modelling the required stabilising moment in order to obtain $FS \geq 1$.

The driving and stabilising moments are calculated using circular slip surfaces. z_{opt} is found as the necessary moment in order to obtain equilibrium in the deterministic design situation. The limit state equation for the railway embankment is defined by the following equation:

$$g(x) = \frac{M_{\text{stab}} + z}{M_{\text{driv}}} \tag{4}$$

where z is an outcome of the stochastic variable modelling the uncertainty related to the stabilising moment, z_{opt}, from the stabilising berms. All parameters in Eq. (4) are outcomes of the individual stochastic variables.

2.2. Stochastic modelling of soil

The undrained shear strength of the soft layer under the embankment is modelled as a stochastic field, taking into account the dependency of overburden stress and correlation between strengths of different positions. The mean value and standard deviation of the undrained shear strength of a given point are

$$\mu_{c_u} = 0.4H_{\text{tot}}\gamma', \quad \sigma_{c_u} = V_{c_u}\mu_{c_u} \tag{5}$$

where H_{tot} (m) is the total height of the soil above the calculation point, γ' (kN/m^3) is the effective unit weight of the soil, and V_{c_u} is the coefficient of variation for the soil. If there are several layers above the calculation point, the multiplication is carried out for each layer separately, thus accounting for the strength increase of soils under the embankment due to consolidation. The mean values of the undrained shear strength c_u modelled by Eq. (5) is illustrated in Fig. 1 for an embankment with the height of 10 m.

The strength is, however, random of nature and therefore local variations occur (Nishimura et al., 2011; Wang et al., 2011). The correlation coefficient of the shear strength is modelled by the following function (JCSS, 2006):

$$\rho_{ij} = \exp\left(\frac{|x_i - x_j|}{d_x} - \frac{|y_i - y_j|}{d_y}\right) \tag{6}$$

where d_x (m) and d_y (m) are the correlation lengths in the horizontal and vertical directions, respectively. They are chosen based on literature study (JCSS, 2006), and on Danish geotechnical experts assessing borehole profiles and vane shear tests from the site (Lodahl et al., 2012). The values of d_x and d_y used in the analysis were 3 m and 1 m, respectively.

The stochastic field modelling the undrained shear strength is discretized and the discretized shear strengths are modelled by the vector, $\mathbf{c}_u$, which in each entry holds the undrained shear strength

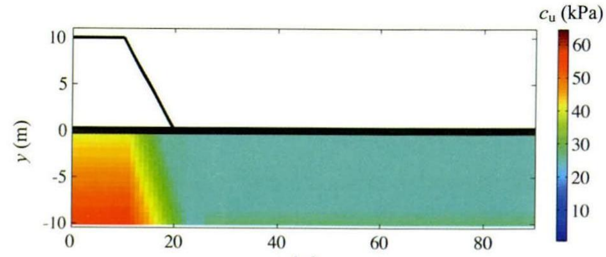

Fig. 1. Mean values of c_u of soft soil deposits under the embankment as modelled by Eq. (5).

of the soil in a point and was modelled by Ayyub and McCuen (2002):

$$\boldsymbol{c}_{\mathrm{u}} = \boldsymbol{\mu}_{c_{\mathrm{u}}} + \boldsymbol{DTU} \tag{7}$$

where $\boldsymbol{\mu}_{c_{\mathrm{u}}}$ is a vector containing the mean values obtained from Eq. (5), as indicated in Fig. 1; $\boldsymbol{D}$ is a diagonal matrix holding the standard deviations of the undrained shear strength at the respective points; $\boldsymbol{T}$ is a matrix incorporating the correlations between two points in the soft layer; and $\boldsymbol{U}$ is a vector with standard normally distributed stochastic variables (mean value and standard deviation equal 0 and 1, respectively).

The diagonal of $\boldsymbol{D}$ contains the standard deviations of the undrained shear strength:

$$D_{ii} = \mu_{c_{\mathrm{u}},\,i} V_{c_{\mathrm{u}},\,i} \tag{8}$$

where the mean value, $\mu_{c_{\mathrm{u}},\,i}$ and the coefficient of variation $V_{c_{\mathrm{u}},\,i}$ are chosen corresponding to the point represented by the position i. $\boldsymbol{T}$ is found using the Cholesky-transformation of the correlation matrix $\boldsymbol{\rho}$ which must be positive definite:

$$\boldsymbol{\rho} = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{T} \tag{9}$$

where $\boldsymbol{\rho}$ is a matrix with correlations between the stochastic variables describing the undrained shear strength at two points (x_i, y_i) and (x_j, y_j), see Eq. (6).

The stochastic field modelling the undrained shear strength is discretized using the midpoint method. The mesh size of the discretised field is approximately 0.5 m. An example of the generation of the stochastic field is presented in Fig. 2.

2.3. Modelling of train load

The train load on the embankment was modelled by an extreme load on the primary track and a simultaneous load on the secondary track. The distribution function for the extreme maximal load was modelled by

$$F_{\mathrm{p}} = \exp\{-[1 - F_X(x)]N\} \tag{10}$$

where F_X is the distribution function of the individual annual maximum load from a single freight train, and N is the number of freight trains per year. The extreme load is illustrated in Fig. 3, along with the distribution of the non-extreme train load, which is applied on the secondary rail.

The secondary train load is modelled as Gaussian distribution. The mean values and standard deviations for both train loads are chosen in order to fit the Danish design codes for railways, yielding characteristic values of 110 kN/m and 175 kN/m for the secondary and primary loads, respectively. The train loads are multiplied with a partial factor of 1.4 and are applied as design loads in the analyses.

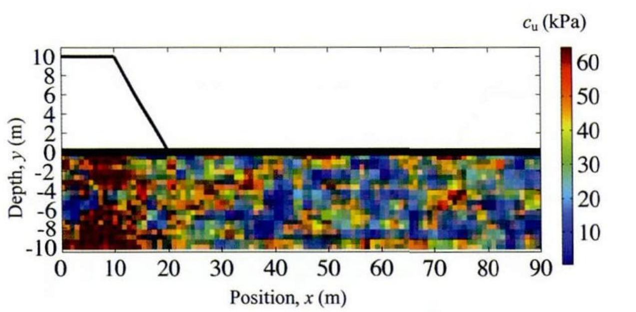

Fig. 2. Realization of the undrained shear strength of the soft layer under the embankment. Note the randomness and the effect of the overburden pressure under the embankment. Correlation lengths $d_x = 3$ m and $d_y = 1$ m.

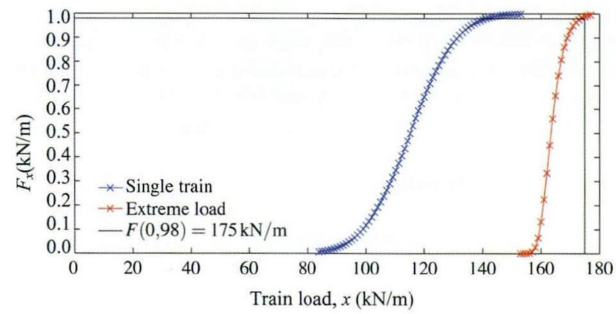

Fig. 3. Distributions of train loads on the embankment.

3. Estimation of reliability by simulation

Since the required reliability of the embankment is high, the number of outcomes needed by a simulation approach to obtain a certain estimate of the probability of failure becomes huge. Using a "brute-force" Monte Carlo simulation technique, the required number of simulations can thus be estimated from $N = 10/P_{\mathrm{f}}$. For the target reliability level, $\beta = 5.2$, $N = 2 \times 10^7$ can be obtained. The crude Monte Carlo sampling method is thus ineffective. That is why asymptotic sampling (Bucher, 2009; Sichani et al., 2011) is used.

Fig. 4 shows the results from a reliability analysis with different embankment heights and a fixed thickness of the soft layer underneath the embankment, $t = 10$ m.

It is seen from Fig. 4 that the reliability index decreases with decreasing partial factor, as expected. The optimal partial factor can be identified as the value *just* making the structure safe enough but not "too" safe. Other thicknesses of the soft layer are tested as well, namely $t = 7.5$ m and $t = 5$ m. The results for these cases are illustrated in Figs. 5 and 6, respectively.

From Figs. 4–6 it is found that the calculated reliability indices indicate that the partial factor can be reduced significantly from the present factor of $\gamma_{c_{\mathrm{u}}} = 1.8K_{\mathrm{FI}}$ to $\gamma_{c_{\mathrm{u}}} = 1.5K_{\mathrm{FI}}$. With $\gamma_{c_{\mathrm{u}}} = 1.5K_{\mathrm{FI}}$, sufficient safety is achieved to satisfy the target reliability index$\beta \geq 5.2$, which is suggested in the Danish design codes for bridges.

Further details on the calibration results can be found in Lodahl et al. (2012).

4. Examples and verification

To verify the validity of the applied slip surfaces, actual embankments designed at two Danish sites were analysed (Lodahl

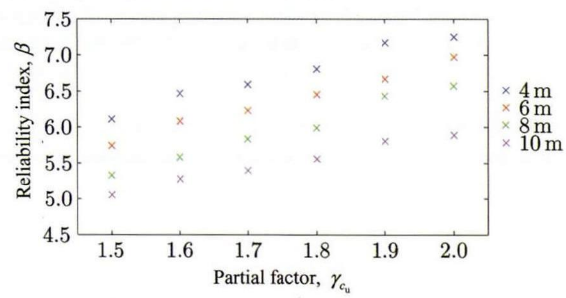

Fig. 4. Reliability index at different partial factors for different embankment heights. Thickness of soft layer $t = 10$ m.

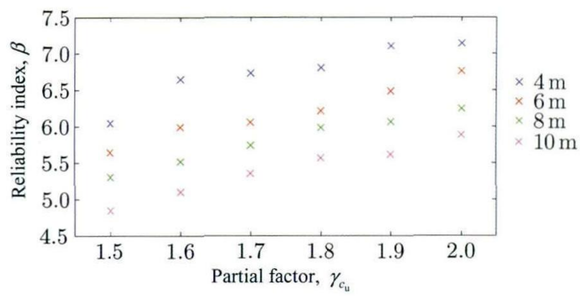

Fig. 5. Reliability index at different partial factors for different embankment heights. Thickness of soft layer $t = 7.5$ m.

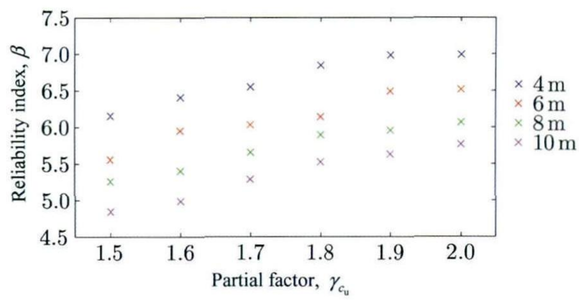

Fig. 6. Reliability index at different partial factors for different embankment heights. Thickness of soft layer $t = 5$ m.

et al., 2013). In the design process for both embankments, the updated partial factors were used and the circular slip surfaces used for the partial factor calibration were compared with the slip surfaces found using *finite element modelling*.

The embankments were designed using the commercial software package SLOPE/w from Geostudio, using circular slip surfaces to determine the necessary size of the stabilising berms for fulfilling the design requirements in the Danish National Annex to Eurocode 7. The results from SLOPE/w were compared with simulations using the finite element tool Plaxis 2D.

In Plaxis the soil was modelled using 15-noded plane strain elements in an unstructured mesh, and using the Mohr–Coulomb material model with characteristic strengths parameters. The initial stress field was set up using a K_0-procedure, after which the embankment was built at intervals. Next, the groundwater level was raised based on the field measurements, and the stabilising berms were modelled. Finally, the train load on top of the embankment was activated, and a φ-c' reduction was carried out.

During this analysis, the strength parameters, the angle of internal friction, φ, and the cohesion, c', were reduced until failure occurred in the embankment, thus the factor describing the extra capacity or safety against failure, M_{sf}, can be found. For the undrained case in cohesive materials, this factor is directly comparable with the partial factors applied during design of the embankments.

4.1. Embankment at Nordvestbanen

At Nordvestbanen in northwestern Zealand, Denmark, an existing railway line was upgraded from one to two tracks. Therefore the existing embankment was expanded with an extra track. Simultaneously, the maximum allowable train velocity was raised to 160 km/h, causing larger loads on the embankment. The embankment is situated on very soft, Holocene organic soils, mostly calcareous gyttja, peat and porous limestone. At the side of the new track, the soft soils below the embankment were replaced with sand, in order to reduce settlements of the new embankment. At the side of the existing embankment, sufficient stability is ensured by stabilising berms.

The embankment is approximately 8 m high, and consists of clay layers interstratified with sand layers. A stabilising berm with a width of 9 m and a height of 2.5 m was found to ensure satisfactory safety. The design was carried out in high consequence class, CC3 (DS/EN, 1997-1, 2007), and thus the applied partial factor was $\gamma_{c_u} = 1.5K_{FI} = 1.65$.

The calculated φ-c' reduction factor from Plaxis 2D yielded $M_{sf} = 1.626$ and the failure mechanism bears close resemblance to the circular slip surfaces from SLOPE/w, see Fig. 7.

4.2. Embankment by the town of Ring

A second case was investigated, namely an embankment by the town of Ring in Zealand, Denmark. At this location, an existing

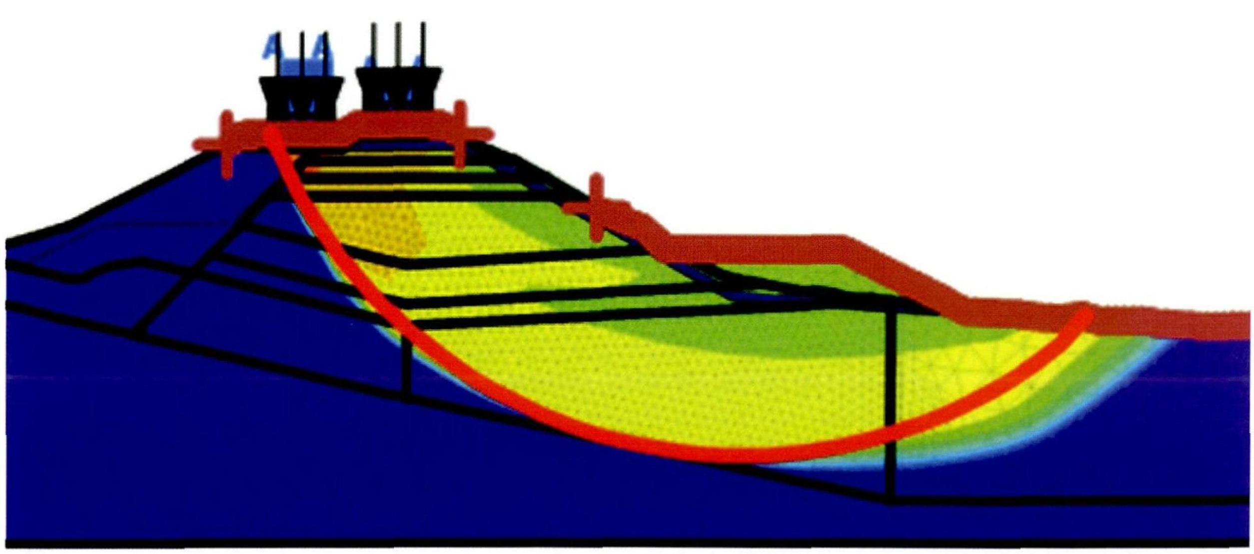

Fig. 7. Embankment at Nordvestbanen with slip surfaces from Plaxis (lighter colours) and SLOPE/w (red line).

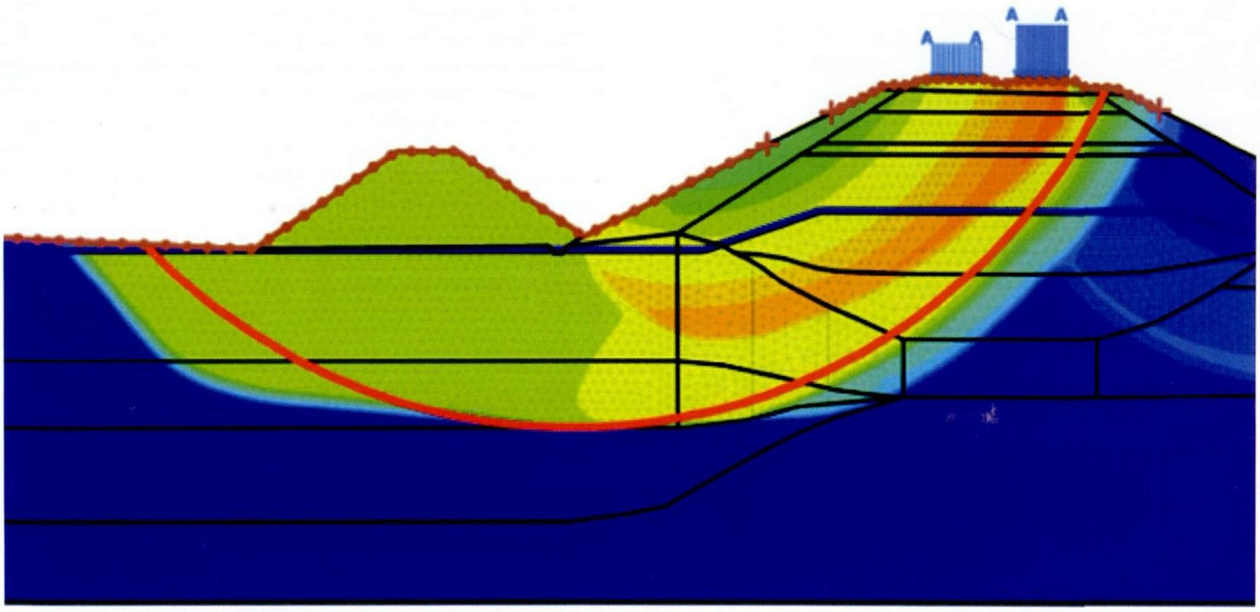

Fig. 8. Embankment by the town of Ring with slip surfaces from Plaxis (lighter colours) and SLOPE/w (red line).

embankment was stabilized with berms due to large settlements. The embankment is 8 m high and situated on top of very soft soils, like gyttja and peat. It is not economically viable to remove the soft deposits, and stabilising berms on both sides of the railway embankment were consequently proposed.

The train load on top of the embankment was not increased, so the embankment was designed to consequence class CC2 (DS/EN1997-1, 2007), which is allowed for existing structures according to the Danish railway codes. The partial safety factor of $\gamma_{c_u} = 1.5$ was therefore applied to the cohesive materials for this project.

The calculated safety based on the φ-c' reduction was $M_{sf} = 1.405$, which corresponds to an approximate 6.8% difference between SLOPE/w and the Plaxis 2D model. The identified failure mechanism from the Plaxis 2D model consisted of circular and linear parts, and was not directly comparable with the slip surfaces from SLOPE/w (Fig. 8). As seen in Fig. 8, the failure mechanism from Plaxis 2D was in fact governed by the stabilising berm, which "forced" the slip surface to exit further away from the embankment than that proposed by the circular slip surface from SLOPE/w.

5. Implications from reliability-based design

Based on the calculated difference between SLOPE/w and Plaxis 2D, a *bias* was added in determination of the stabilising moments in Eq. (3) in the probabilistic analyses. This bias accounts for systematic overestimation due to the application of circular slip surfaces, and was suggested to be 10% in the analyses. Using the bias, the probabilistic analyses were re-evaluated, yielding a lower but still acceptable level of safety in consequence class CC2 (Eurocode 7).

Modifying the partial factor concerning undrained shear strength significantly reduced the costs of upgrading the embankments at Nordvestbanen, where the contractor estimated that approximately €13.5 × 10^6 was saved on the earth works.

At the embankment at the town of Ring, the savings are estimated to approximately €165,000 in total. This sum may seem insignificant, but will increases 55% of the budget for earth works if the partial factor required in the Danish National Annex to Eurocode 7 was to be used.

6. Conclusions

Based on the work of Lodahl et al. (2012), a probabilistic design method for railway embankments has been presented. The method is based on the assumption of circular slip surfaces in the undrained case for cohesive soils and has shown that an acceptable safety level is reached even for a lower partial factor than that stated in the Danish National Annex to Eurocode 7.

A comparison between a finite element solution and a design using circular slip surfaces has been presented. Generally a good agreement is found between the finite element solution and the circular slip surface; however, stabilising berms have a tendency to force the slip surface to exit the soils further away from the embankment than that predicted by the circular slip surfaces.

A difference was found between the calculated safety from the φ-c' reduction and the design safety using circular slip surfaces. This was accounted for by introducing a bias in the calculations, which was fitted to the actual case, and analysing the difference between the safety for a circular slip surface and the safety found in Plaxis.

In total, the probabilistic design method is considered to be a very effective tool, enabling the designer to specify the desired safety level and to design the structure accordingly. Thus, applying the probabilistic design method, cheaper designs are often found with great economic benefit to the client.

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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