# Three-Dimensional Limit Equilibrium Slope Stability Benchmarking

**M.D. Fredlund** *SoilVision Systems Ltd., Saskatoon, Canada*

**H. Lu** *SoilVision Systems Ltd., Saskatoon, Canada*

**D.G. Fredlund** *Golder Associates, Saskatoon, Canada*

## Abstract

*Slope stability analyses have been performed using various software packages on a routine basis in geotechnical engineering since the 1980s. A variety of 2D software packages have been developed at the academic level and the commercial level to provide a comprehensive analysis of slopes. Many of these packages are now used by geotechnical consultants, often with little regard for the potential differences between the various analysis methods implemented in different software packages. Many of the early software packages do not appear to have been tested against a significant number of comprehensive benchmark examples. Recent studies have resulted in the collection of an extensive library of benchmark problems to which slope stability software packages can be referenced. In most of these situations a "correct" solution can be presented which provides a "reference benchmark" for testing slope stability software packages. The variation is typically not quantified. This paper summarizes the results of an extensive comparison of benchmark results obtained during the development of the SVSLOPE-3D software package. Issues surrounding the correct solution of benchmarks are discussed. The potential applications of this technology will also be discussed.*

## 1 Introduction

Slope stability problems in geotechnical engineering involve the solution of equilibrium equations of force and moment. This is traditionally accomplished through the method of slices techniques or more progressive stress-based methods.

It is current industry practice is to perform slope stability analyses in 2D. This makes several fundamental assumptions, chiefly that the slope is homogenous across the width and that the slip surface is inherently cylindrical in nature. Gitirana et al. 2008 found that analyzing problems in 3 dimensions can lead to differences in the lowest Factor of Safety between 15% and 50%. It is prudent for engineers to consider the 3D scenario. With the advent of 3 dimensional slope stability modeling more complex scenarios can be considered with more confidence in the results, due to a greater understanding of slope stability analysis.

This paper presents some of the extensive three dimensional benchmarking which has been performed on the SVSLOPE software package. Comparisons are made to literature scenario, as well as to other software.

### 1.1 Reasons for Verification

"Verification" is generally achieved by solving a series of so-called "benchmark" problems. "Benchmark" problems are problems for which there is a closed-form solution or for which the solution has become "reasonably certain" as a result of longhand calculations that have been performed. Benchmarks can be drawn from historical failure scenarios, literature studies, as well as through comparisons to other software. Publication of the "benchmark" solutions in research journals or textbooks also lends credibility to the solution. It must always be remembered there is never such a thing as complete software verification for "all" possible problems. Rather, it is an ongoing process that establishes credibility with time.

# 2 Overview of Slope Stability Modeling

Figure 1 provides an overall classification of slope stability methods of analysis. It can be seen that a distinction should be made between analysis methods and searching techniques. In 1977, Fredlund and Krahn classified the limit equilibrium methods of slices according to the elements of static equilibrium that were satisfied when solving for the factor of safety. This included categorizing the assumptions used to render the analysis determinate.

In 1981, Fredlund, Krahn and Pufahl further extended the comparison of slope stability method of slices to include additional methods (Fredlund et al, 1981). Most of the limit equilibrium methods of slices made an assumption regarding the interslice forces (e.g., the interslice force function). Consequently, most of the methods of slices differed in the manner by which the normal force at the base of a slice was calculated. Common to all the methods of slices was the manner in which the factor of safety was defined and the fact that the normal force was computed from statical considerations of one slice through a potential sliding mass.

Figure 1 shows that it is now possible to also take into consideration the search technique associated with the determination of the shape and location of the critical slip surface. The finite element stress analysis method can also be used to determine the normal force at the base of a slice, giving rise to the Enhanced (Kulhawy) Limit method as well as other optimization techniques (e.g., Dynamic Programming).

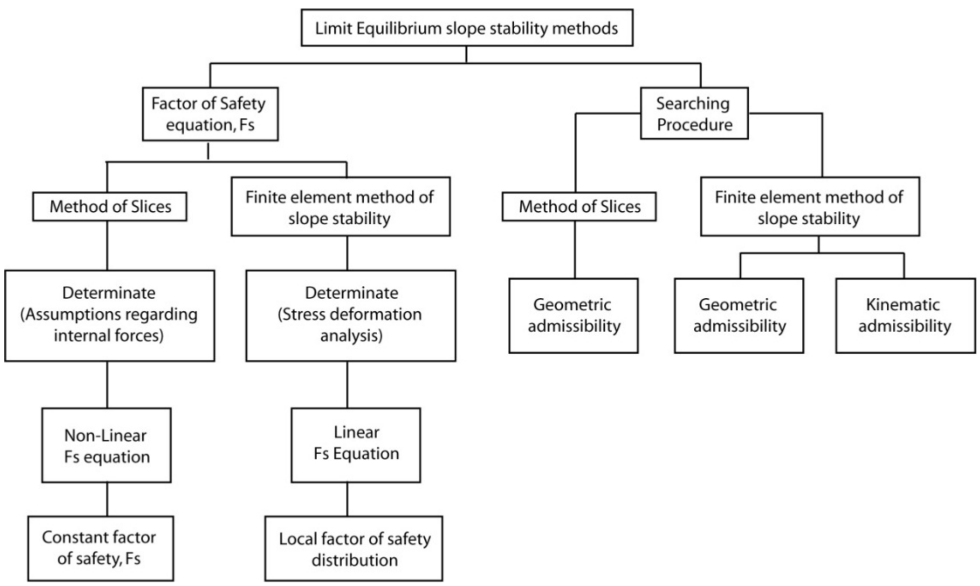


Figure 1. Overall classification of slope stability methods of analysis

## 2.1 3D Slope Stability

In extending slope stability analyses into 3 dimensions, the "method of slices" becomes the "method of columns." When considering a "slice" out of a slope, only the 2 dimensional inter-slice forces are considered, however, in a column, the lateral forces also come into effect, creating a much more realistic scenario. This requires handling of far more variables; however, the same general slope stability formulations that are commonly performed in 2D (Bishop, Morgenstern-Price, Janbu, etc.) can also be extended into 3D.

Similarly to the circular sliding surfaces commonly found in 2D analyses, an ellipsoidal sliding surface can be defined. Other shapes are also possible when considered in 3D, such as multi-planar wedges and fully specified sliding surfaces along known failure zones. The software also allows other features to be considered such as external loads, supports, bedrock (slip surfaces cannot pass through these zones), tension cracks and discontinuities.

# 3 Benchmarking

In the following sections some of the benchmarks used to test the SVSlope 3D solution engine are presented.

## 3.1 Ellipsoidal Sliding Surface with Toe Submergence

This problem models upstream of an earth fill dam. The dam is built with a sloping clay core surrounded by granular material. The water surface is higher than the ground surface at the toe of the dam, so toe submergence will be considered. The model was published in the CLARA/W manual (Hungr, 2010). Comparisons will be made to the CLARA/W predictions.

### *3.1.1 Geometry and Material Parameters*

The pore-water pressure is defined with the water surface grid data. To allow for direct comparison to CLARA/W, a single slip surface was defined. An ellipsoidal surface was used, where the center of the ellipsoid is defined at (149.170, 0, 356.090) and the tangent plane is located at 108.40. Since the model is symmetrical, only half of the slope is analyzed. The geometry and material parameters are shown in Figure 2 and Table 1. The model geometry is extruded uniformly in the third dimension.

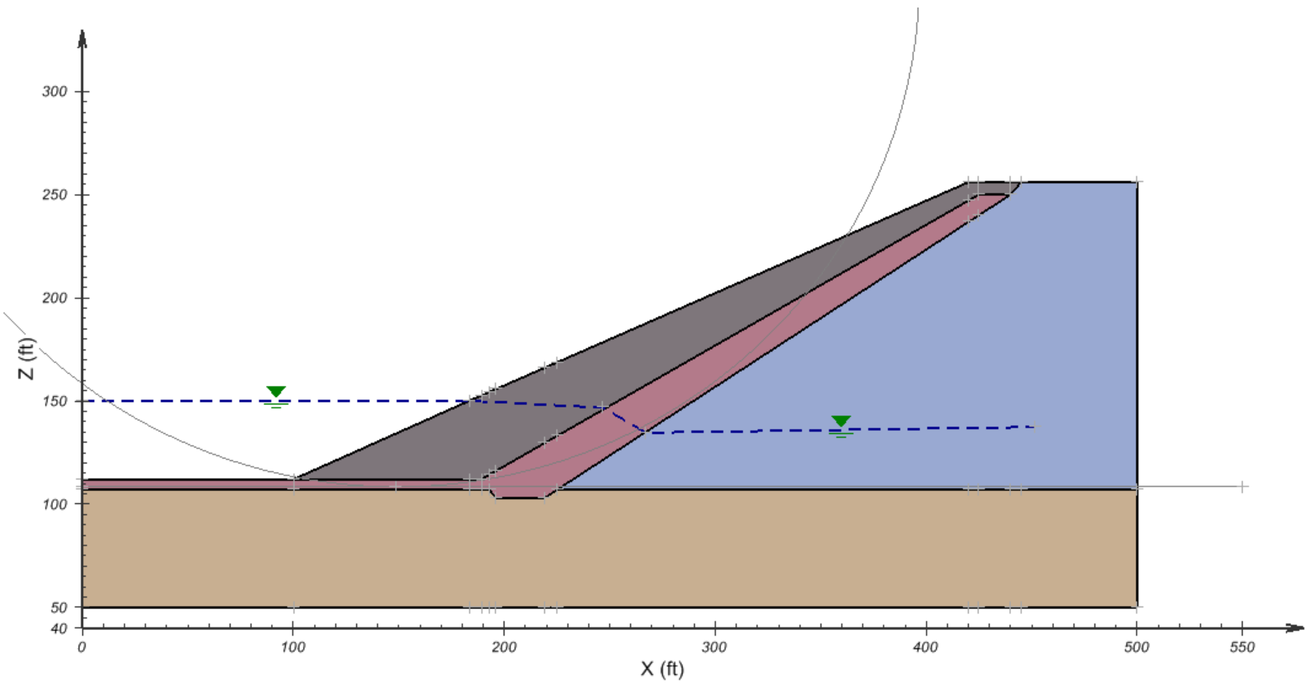


Figure 2. Geometry of the Ellipsoidal Toe Submergence model with the water surface

Table 1. Material Parameters of the Ellipsoidal Toe Submergence

| | c (psf) | ϕ (degrees) | γ (lb/ft^3) |
|---|---|---|---|
| RockFill | 0 | 35.0 | 70.6 |
| Core | 100 | 29 | 70.6 |
| Fill | 0 | 28 | 70.6 |
| R1 | 10000 | 35.0 | 100 |

### *3.1.2 Results and Discussions*

The results of the analysis are presented in Table 2 and Figure 3. It can be seen that the results of the software match reasonably well with CLARA/W. Differences of less than 5% are considered reasonable.

Table 2. Results of the Ellipsoidal Toe Submergence model analysis

| | Factor of Safety | | | Difference |
|---|---|---|---|---|
| Method | CLARA/W | SVSLOPE 3D | | (%) |
| | | Moment | Force | |
| Bishop | 1.300 | 1.273 | | 2.104 |
| Janbu Simplified | 1.230 | | 1.208 | 1.770 |
| Spencer | 1.260 | 1.213 | 1.213 | 3.708 |

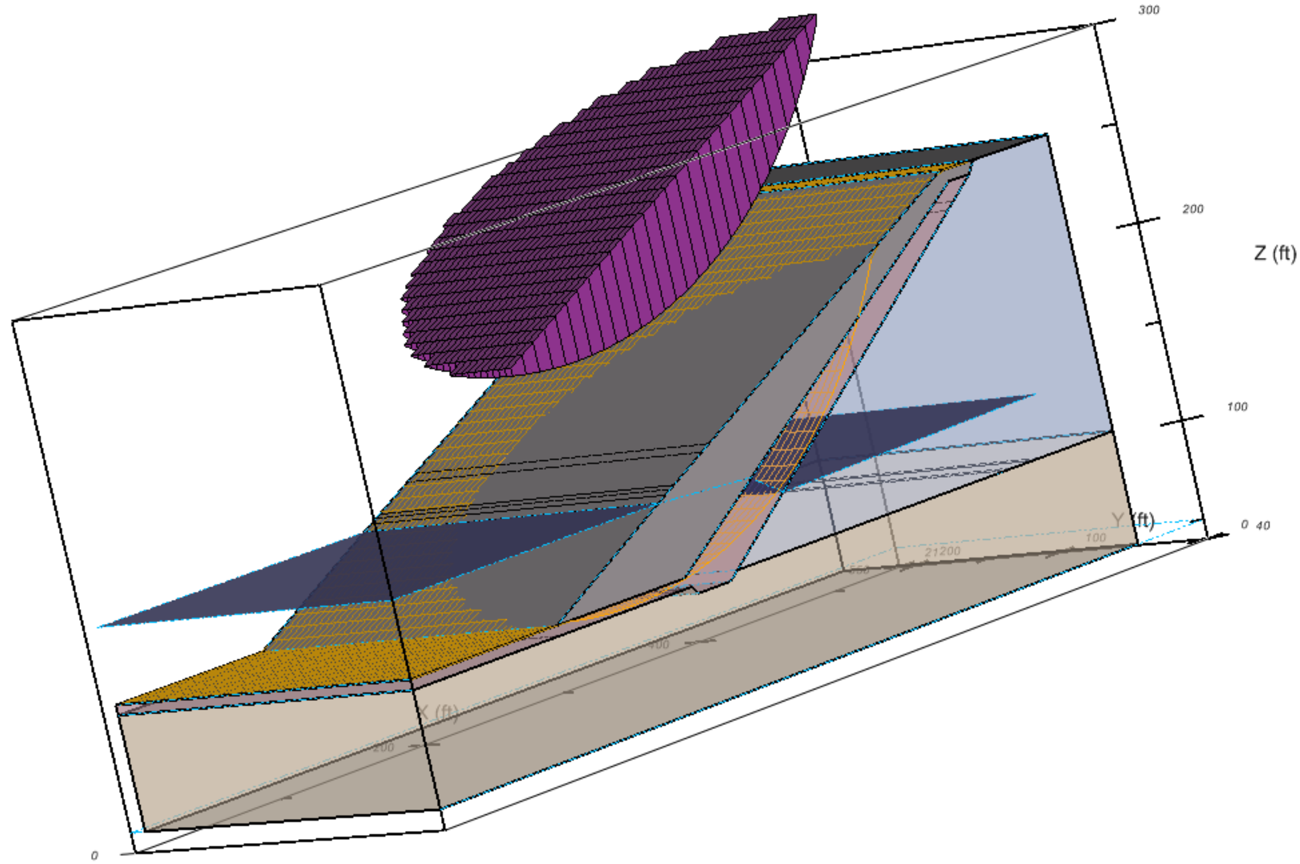


Figure 3.  Result of analysis of Ellipsoidal Toe Submergence model with display of 3D sliding mass

## 3.2 A Multi-Planar Wedge Surface

This model demonstrates a multi-planar wedge sliding surface. It is applied to a waste pile where there is a weak interface between the waste and foundation materials. The weak surface is defined using a discontinuity material "disc". The other three wedge planes forming the sliding surface have the properties of the waste material.  This procedure is applicable to cases where the use of geotextiles is being examined.

### *3.2.1 Geometry and Material Parameters*

Pore-water pressure is present in this example and is defined by a water surface at the interface between the waste and foundation. The corresponding model and wedges data are presented in Figure 4. The material parameters are presented in Table 3.

Table 3 Material Parameters of the Multi-Planar Wedge model

| | c ($kN/m^2$) | $\phi$ (degrees) | $\gamma$ ($kN/m^3$) |
|---|---|---|---|
| Fill | 0 | 35 | 18 |
| Clay_Foundation | 50 | 20 | 20 |
| disc | 0 | 12 | 0 |

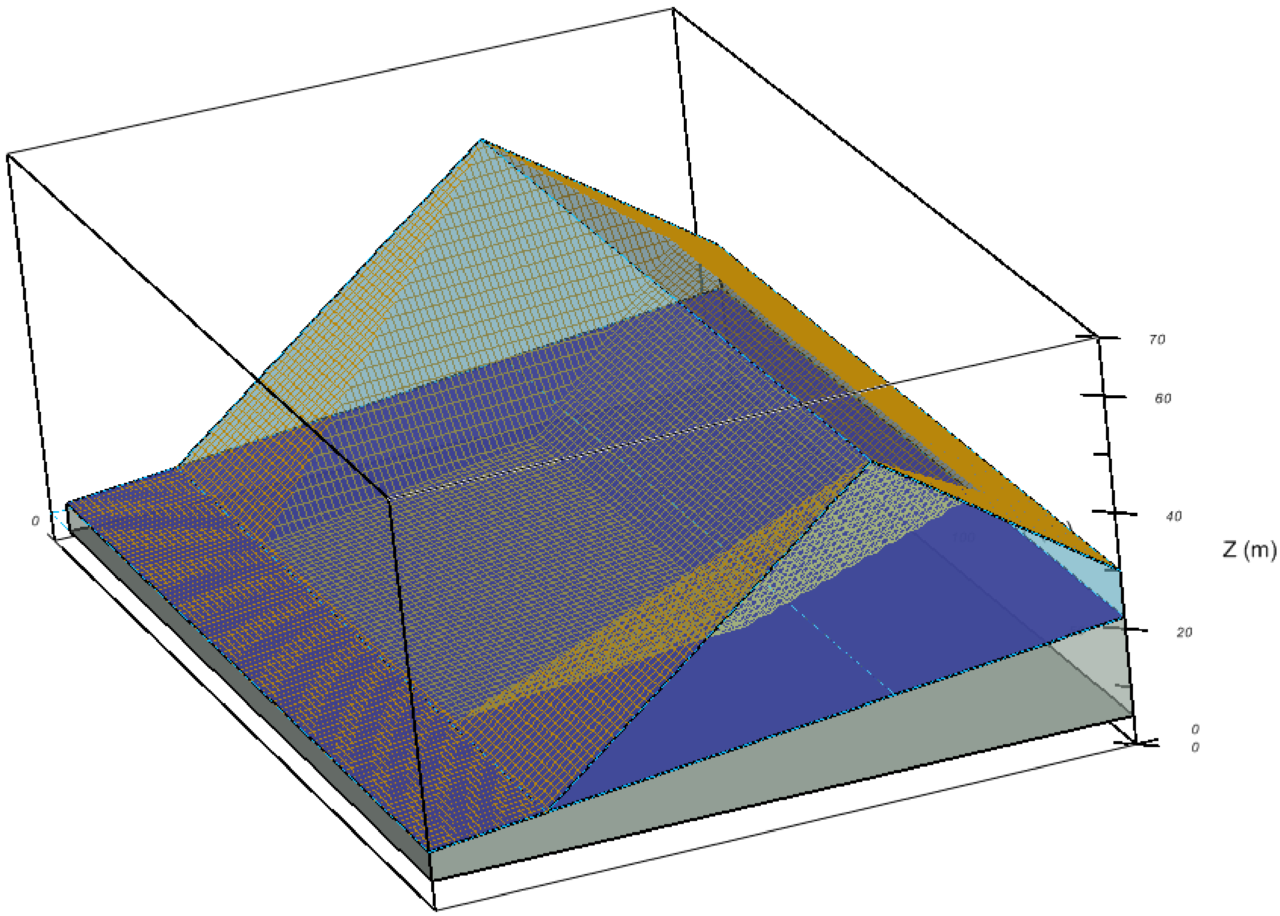


Figure 4. Geometry of Multi-planar wedges model with the wedges data

Table 4: Wedge Sliding Surfaces

| | X (m) | Y (m) | Z (m) | Dip (deg.) | Dip Dir. (Deg) |
|---|---|---|---|---|---|
| Wedge #1 | 0 | 90 | 10 | 7 | 0 |
| Wedge #2 | 60 | 90 | 12 | 32 | 0 |
| Wedge #3 | 0 | 90 | -35 | 45 | 87 |
| Wedge #4 | 0 | 90 | -35 | 45 | -87 |

### *3.2.2 3.2.2 Results and Discussions*

The results are shown in Table 5 and Figure 5. Good agreement is observed for the Janbu Simplified and Bishop methods. CLARA/W does not have a converged solution for Spencer method and there is a significant difference between CLARA/W and SVSLOPE 3D with the Morgenstern-Price method. The calculations of the Morgenstern-Price method in the Clara/W software appear questionable given the significant difference between the Bishop and Morgenstern-Price result in the CLARA/W software.

Table 5 Results of Multi-planar wedges

| Method | Factor of Safety | Difference |
|---|---|---|

| | CLARA/W | SVSLOPE3D | (%) |
|---|---|---|---|
| Bishop | 1.150 | 1.147 | 0.229 |
| Janbu Simplified | 1.150 | 1.149 | 0.128 |
| Spencer | - | 1.172 | - |
| Morgenstern-Price | 1.380 | 1.172 | 15.053 |

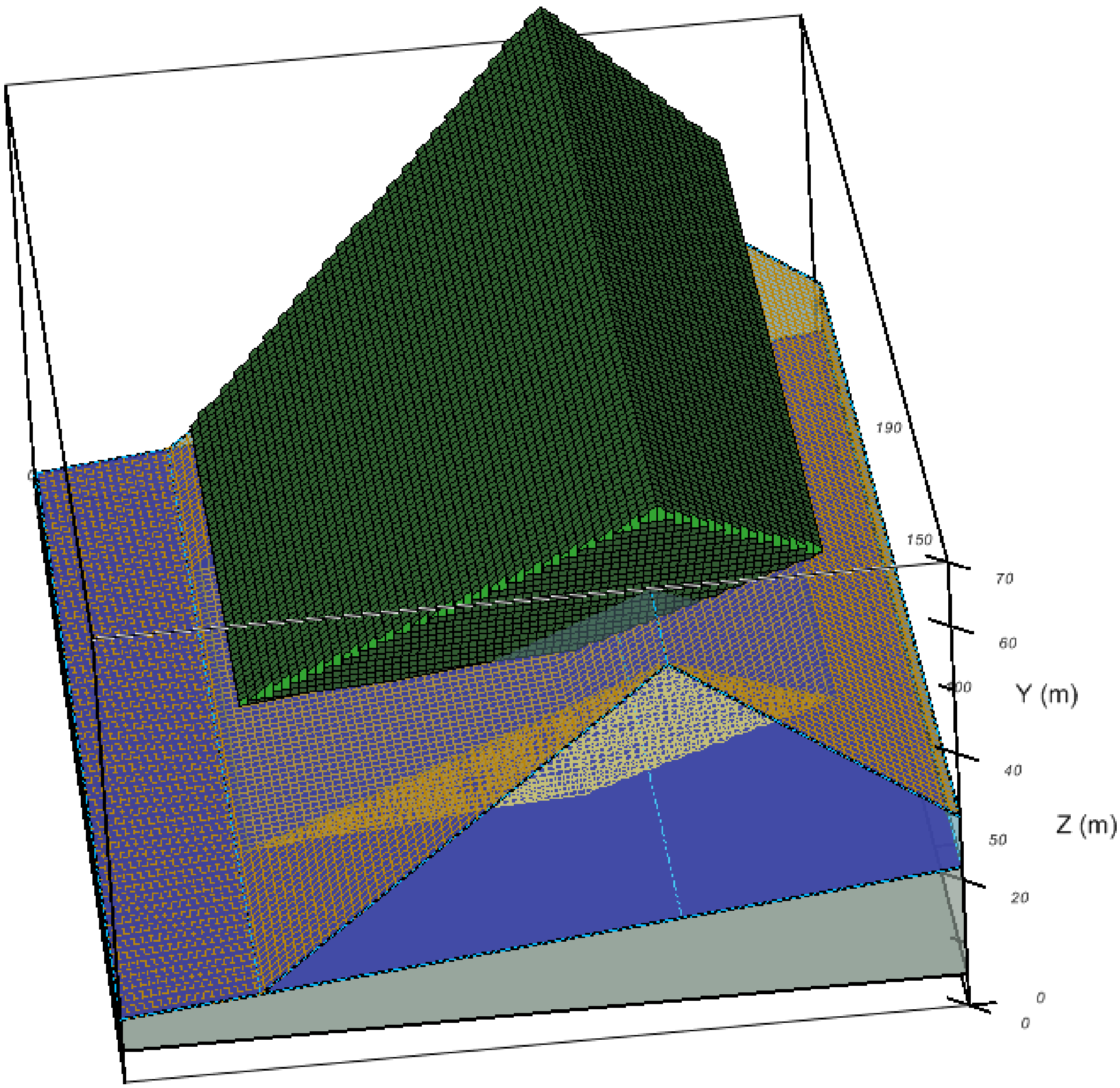

Figure 5. Result of Multi-planar wedges model

### 3.3 Fredlund and Krahn (1977) 2D TO 3D

This model was created based on the 2D example model by Fredlund and Krahn (1977) by extending the 2D model into 3D. This model is a more "exact" match with the original 2D model as the wedge plane and discontinuity material method is not used. In this example an ellipsoidal sliding surface is utilized and the weak layer is kept as a separate layer.

### *3.3.1 Geometry and Material Parameters*

This model consists of a 35 meter high clay embankment with a sloping water table, which exits at the toe of the embankment. The embankment soil is underlain by a thin weak layer of soil. Underlying this is bedrock. The ellipsoidal sliding surface will pass through the bedrock meaning that shearing will occur at the interface between the bedrock and the weak layer. The geometry and material parameters are shown in Table 6 and Figure 6.

Table 6 Material Parameters of Fredlund and Krahn (1977) 3D model

| | **c (psf)** | **$\phi$ (degrees)** | **$\gamma$ (lb/ft^3)** |
|---|---|---|---|
| clay | 600 | 20 | 120 |
| Weak layer | 0 | 10 | 120 |
| Bedrock | | | |

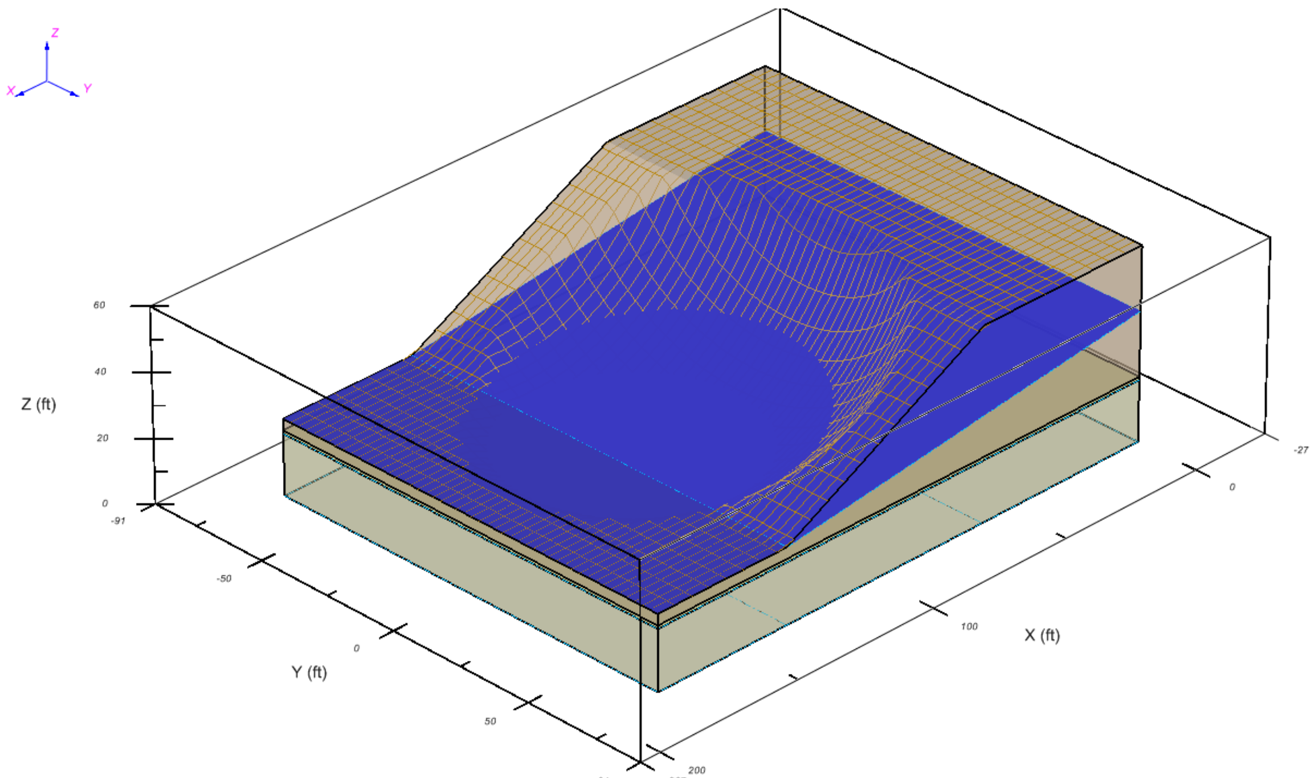


Figure 6. Geometry of 3D Example Model – Fredlund and Krahn (1977)

Table 7: Ellipsoidal Sliding Surface Coordinates

| | |
|---|---|
| X-Coordinate | 120.000 |
| Y-Coordinate | 0.000 |
| Z-Coordinate | 90.000 |
| Tangent Plane | 10.000 |
| Aspect Ratio | 1.000 |

### *3.3.2 Results and Discussions*

Figure 7 illustrates the 2D results from Fredlund and Krahn (1977) compared to the SVSLOPE 3D results and CLARA/W results. The 3D FOS is about 30% larger than 2D FOS on average. The higher factor of safety observed when comparing 3 dimensional analyses to 2 dimensional analyses have been observed in other completed studies (Gitirana, Santos, & Fredlund, 2008).

Table 8 Results of the Fredlund and Krahn (1977) 3D model

| **Method** | **Factor of Safety** | | | **Difference (%)** |
|---|---|---|---|---|
| | **Fredlund and Krahn (1977) 2D** | **CLARA/W** | **SVSLOPE 3D** | |
| Ordinary | 1.171 | | 1.487 | - |
| Bishop Simplified | 1.248 | 1.62 | 1.638 | 1.108 |
| Janbu Simplified | 1.333 | | 1.622 | - |
| Corps. of Engineers #1 | - | | 1.840 | - |
| Corps. of Engineers #2 | - | | 1.742 | - |
| Lowe-Karafiath | - | | - | - |
| Spencer | 1.245 | | 1.682 | - |
| Morgenstern-Price | 1.250 | | 1.647 | - |
| GLE | - | | 1.648 | - |
| Sarma | - | | 1.675 | - |

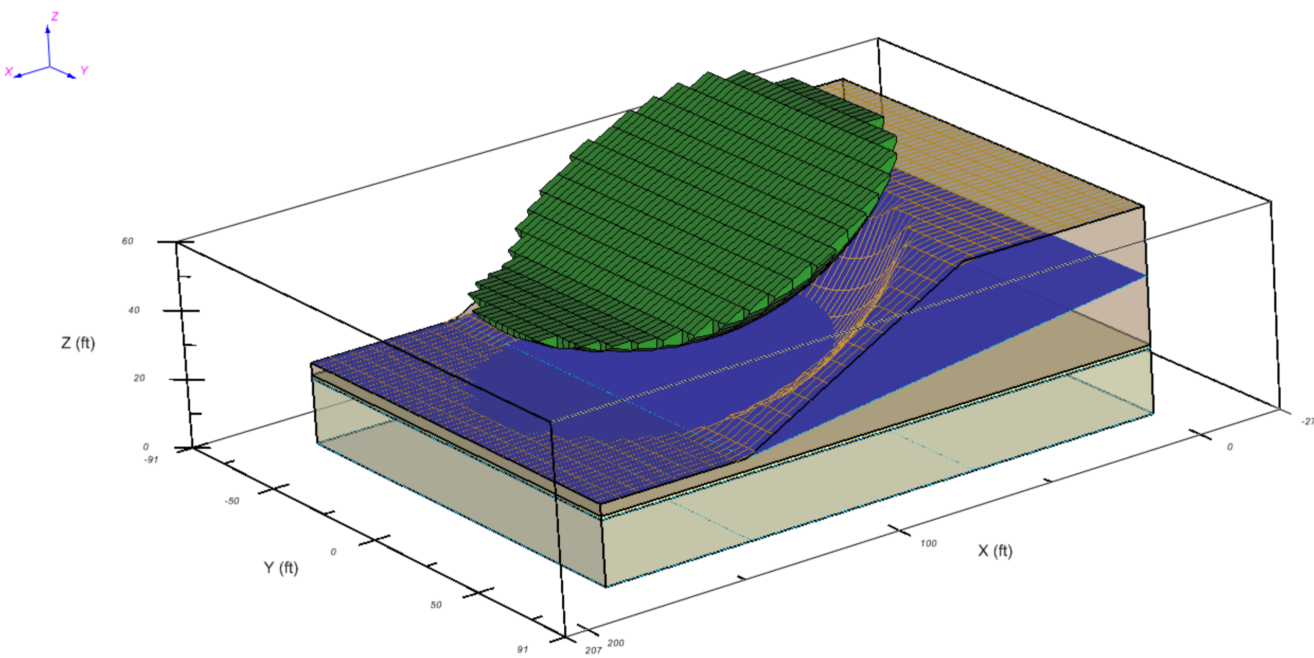


Figure 7. Results of the Fredlund and Krahn (1977) 3D model

# 4 Potential Applications

Three dimensional slope stability analyses have many potential applications. Any application where the geometry is too complicated to be approximated in two dimensions will benefit from three-dimensional implementation. Some problems are inherently three dimensional, such as:

- **Levee analysis:** Levee systems can extend over many miles. This makes transportation and placement of the earth used for construction generally quite costly. Most existing designs are based on a 2D analysis; however this may lead to the levee structures being over-designed and result in increased costs to construct.
- **Pit slope stability:** The highly 3D nature of mining pits means that they often require a 3D analysis. The limit equilibrium method is well-suited for the highly irregular geometry often encountered in this application.
- **Earth storage:** Often it is necessary in the mining industry to store borrow materials, such as top soil used for reclamation, for long periods of time. By stacking the material higher and making the slopes steeper the area required for storage can be reduced. A 3D analysis allows the calculation of more realistic factors of safety and allows the opportunity to design storage facilities which optimize design.
- **Tailings and mine rock piles:** The land requirement for the storage of tailings and mine rock piles can be costly. An optimized design performed using a 3D analysis allows for the storage of more tailings per unit area.
- **Heap leach analysis:** Larger amounts of ore can be extracted on a heap leach pile with steep side slopes. However, the increased side slope steepness results in a condition closer to failure. A 3D analysis can be used to more closely represent the geometry of the heap leach pile. It is important to accurately balance the slope angle design and the ore recovery.

# 5 Conclusions

Comprehensive verification of engineering software is critical to application of new methodologies and confidence related to various new and non-standard scenarios. Development of a new software package has required a significant benchmarking effort. This paper presents 3 of these benchmarks that have been created to validate the use of the three dimensional, slope stability software in geotechnical engineering. Other verification examples are published in the SVSlope verification manual available on the SoilVision System Ltd. website.

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