3D SHEAR STRENGTH REDUCTION FINITE ELEMENT ANALYSIS

H. H. LU
SoilVision Systems Ltd., Saskatoon, Canada, hailu@soilvision.com

L. M. XU
SoilVision Systems Ltd., Saskatoon, Canada, xu@soilvision.com

M. D. FREDLUND
SoilVision Systems Ltd., Saskatoon, Canada, murray@soilvision.com

D. G. FREDLUND
Golder Associates Ltd., Saskatoon, SK., Canada, del_fredlund@golder.com

Limit equilibrium method (LEM) analysis of slope stability has been commonplace in geotechnical engineering for many years. The factor of safety calculated by a LEM analysis remains an industry standard of design. The Shear Strength Reduction (SSR) technique using finite element method (FEM) analysis to predict a factor of safety for slopes has also been in existence for several decades. Recent software tools allow the improved analysis of 3D slope stability through the LEM and SSR techniques. The advantages of a 3D SSR analysis over 3D LEM analysis are that stresses and deformations can be more accurately accounted for in a slope stability analysis. Assumptions are not needed regarding the shape, location and sliding direction of the failure surface. The purpose of this paper is primarily to compare 3D FEM-SSR stability analysis with 3D LEM analysis through the examination of benchmark examples. In general, well-matched results are obtained between these two methods.

INTRODUCTION

The 2D LEM is widely used in geotechnical engineering for slope stability analysis. However, all slope failures are 3D in reality. The 2D approach is generally considered to be conservative in that 3D influences of geometry are not accounted for in such a 2D analysis. This conservatism has fundamentally been viewed as a "buffer" or added factor of safety but designs that push the limits of analysis can be problematic. Furthermore, the assumption that 2D analyses lead to conservative factors of safety (FOS) is correct only when a pessimistic section of the 3D model is selected for the 2D analyses. The pessimistic section may not be intuitively obvious for some general slopes, (Griffiths *et al.* [2]). It is time consuming to ensure that the 2D section model is the critical, pessimistic 2D section. The Shear Strength Reduction (SSR) technique using FEM analysis to predict a factor of safety for slopes has been in existence for several decades. With the remarkable increase of computing power and the low price of computers, more researchers have given consideration to the use of 3D SSR investigation of slope stability, (Griffiths *et al.* [2]; Wei *et al.* [6]).

However, 3D SSR analysis and 3D LEM analysis are seldom performed in the engineering practice. Recently, the SoilVision Systems has incorporated the 3D FEM-SSR into its commercial SVSOLID 3D package and the 3D LEM into its SVSLOPE package. Consequently, the 3D SSR and 3D LEM are now readily available to geotechnical engineering practice.

SSR FEM FOR SLOPE STABILITY ANALYSIS

The factor of safety (FOS) for a SSR analysis is defined as the ratio of the shear strength of the soil to the shear stress developed along the critical failure surface.

$$c_f = \frac{c}{SRF}$$

$$\emptyset_f = \tan^{-1}(\frac{\tan \emptyset}{SRF})$$

where c and $\emptyset$ are the cohesion and angle of internal friction for the Mohr-Coulomb shear strength parameters. c_f and $\emptyset_f$ are factored shear strength parameters. *SRF* is called the strength reduction factor. In order to reach the state of limiting equilibrium, the *SRF* is gradually increased. This means that the soil shear strength becomes weaker, until it is no longer possible for the FE model analysis to reach convergence. At this stage, it can be said that failure of the slope occurs and FOS = *SRF*. Non-convergence within a specified number of iterations and tolerance is an indicator of slope failure because of the absence of force equilibrium, (i.e. stress and displacement distributions that satisfy the equations of equilibrium cannot be established based on the factored set of shear strength parameters).

The SSR FEM analysis has been shown to be a powerful and useful alternative to conventional LEM slope stability analysis technique. The advantages of the 3D SSR over 3D LEM can be listed as follows: (Griffiths *et al* [2, 3]):

1. SSR can more reasonably account for different boundary conditions and the geometry in the third direction.
2. No assumption needs to be made in advance regarding the shape, location and sliding direction of the 3D failure surface.
3. No assumptions regarding the inter-column forces are required as are needed in the 3D LEM since there is no concept of columns in the SSR FEM analysis.
4. Slope stress and deformation information can be considered and obtained, so it can monitor progressive failure with the increase of the SRF up to the shear failure of the slope.

EXAMPLES

The following examples are provided in order to illustrate and compare the differences between the LEM 3D method as well the FEM-SSR 3D method.

Example 1 - A 3D model with weak layer and pore-water pressures

Fredlund *et al* [1] showed an example that has been used to compare different methods of slices. This model as shown in Figure 1 has been extruded to 3D models and investigated by various researchers, (Zhang [7]; Lam *et al* [5]; Huang *et al* [4]; Griffith *et al* [3]). The 2D model geometry, and pore-water pressure conditions are presented in Figure 1. In present study, it corresponds to case 6 in Fredlund *et al* [1]'s six different cases. The model has a weak layer in the middle, the bottom layer is a bedrock layer, and a piezometric surface is applied to the top two layers.
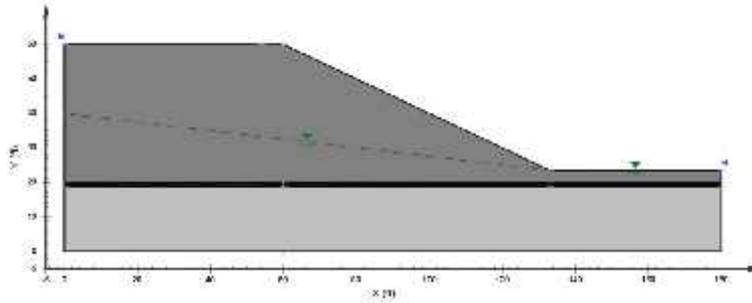
Figure 1. 2D sectional view for the example 2 model

The result of the 3D SSR analysis based on SVSOLID 3D is shown in Table 1 with some other results from Zhang [7] and SVSLOPE 3D. The 2D FOS for the model is 1.248 (Simplified Bishop method) and 1.250 (Morgentern-Price method) from Fredlund *et al* [1]. It can be seen that the 3D FOS is about 15% percentage higher than 2D FOS for this model. The 3D contour of the total displacement at the final stage is shown in Figure 2. Figure 3 shows a y-direction view of the 3D contour of total displacement. The results show the shape of the slip surface which passes though the weak layer above the bedrock layers.

Table 1. Comparison of 3D FOS for the 3D slope model in Example 1

Zhang [7] (LEM)	SVSlope 3D (LEM)	SVSolid 3D (SSR)
1.441	1.453 (Simplifed Bishop); 1.510 (Spencer)	1.446
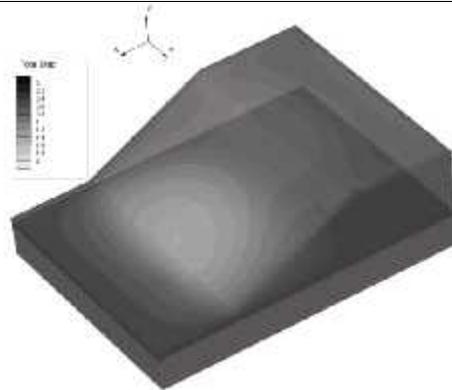
Figure 2. Contour of total displacement of example 2 model at the final stage
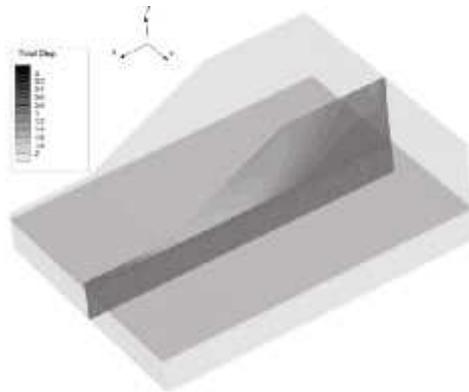
Figure 3. Y-section view of the contour of the total displacement for example 2

Example 2 - A general slope surface

Figure 4 shows a general slope which is located in Northern Alberta, Canada. There is a highway passing through the middle of the model area. The slope material is mainly native clay with cohesion of 1 kPa, angle of internal friction of 15 degree, and a unit weight of 20 kPa.

Figure 4. 3D view of the general slope

There is no obvious sliding direction that can be pre-described for this type of geometry. Both 3D SSR and 3D LEM analysis are performed using SVSOLID 3D and SVSLOPE 3D for this example. The SSR analysis does not require in advance that the direction of the critical slip direction be known. In order to analyze the stability of the slope using 3D LEM, SoilVision Systems has incorporated a new feature that allows for the search for the critical slip surface direction in SVSLOPE 3D. Table 2 shows the comparison of the FOS results based on these two different methods. It can be seen that a close match of the results is obtained. With 3D LEM analysis, the critical slip direction is shown to be 20.43 degrees counter-clockwise (CCW) along positive x-direction. Figure 5 and Figure 6 show the 3D SSR result with contouring of total displacement corresponding to the final stage. Figure 7 shows the 3D LEM results.

Table 2. Comparison of 3D FOS for the 3D general slope model

SVSLOPE 3D (LEM)	SVSOLID 3D (SSR)
1.450 (Simplified Bishop) with critical slip direction angle = 20.43 degree (CCW)	1.491
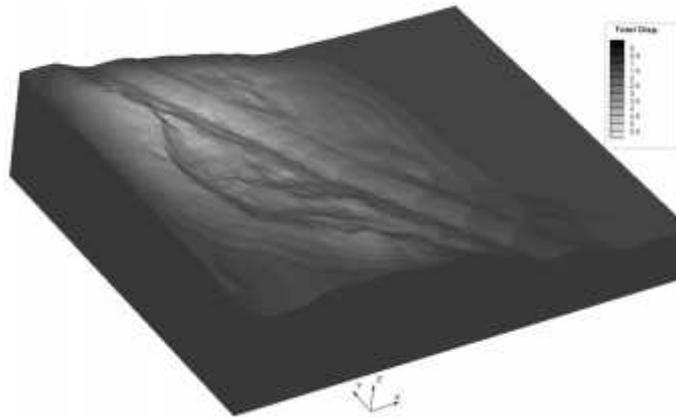
Figure 5. Contour of total displacement of the general slope at the final stage
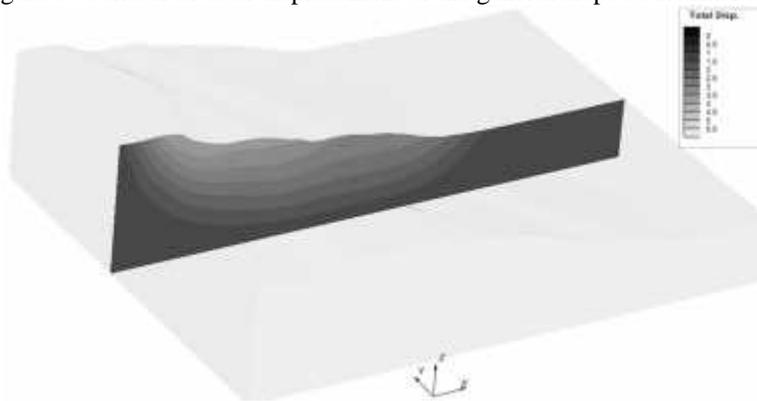
Figure 6. Y-section view of the total displacement of the general slope at the final stage
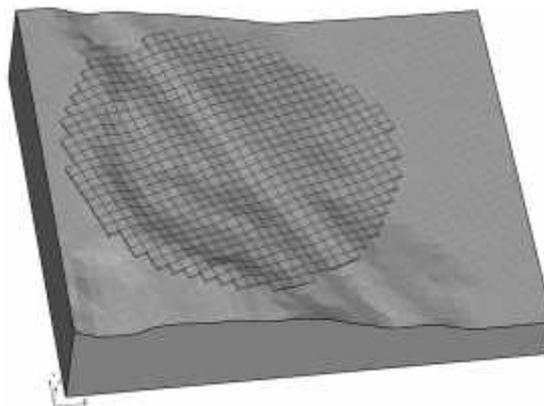
Figure 7. Critical slip mass of the 3D LEM analysis based on SVSLOPE 3D

CONCLUDING REMARKS

The FEM SSR technique has shown increased promise over the past few years as a reasonable methodology for performing slope stability analysis. While the FEM SSR method typically takes more computational power to perform it avoids the complex searching algorithms required to determine the critical slip surface. There are also no restrictions related to the shape of the slip surface in the FEM-SSR methodology. Both of these features make the SSR method potentially advantageous when performing a 3D slope stability analysis. In a 3D analysis both the shape and direction of failure must be reasonably determined to be confident the critical slip surface has been found. The FEM-SSR 3D method also allows the introduction of complex geometries, boundary conditions, and accounting more rigorously for complex material constitutive behavior.

The second example in particular highlights the application of both 3D methodologies to a complex but real-world application in the field of transportation. The accounting of complex geometries in the numerical modeling process is demonstrated.

The current paper has presented the results of two typical examples to demonstrate the close practical agreement between the 3D LEM and FEM-SSR methodologies in typical examples. Differences in the calculated factor of safety between the two methodologies were small. Therefore the potential advantages to performing a FEM-SSR 3D analysis are highlighted.

The use of 3D analysis is more realistic and leads to improved accuracy of the problem at hand. The use of the FEM-SSR 3D methodology can also provide further insight into failure mechanisms as the potential slip surface shape is not specified by the user.

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