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A Computational Fluid Dynamics model of a 20Kg Induction stirred laboratory scaled ladle

Abstract Over the years numerous Computational fluid dynamics models [1,2,3,4,5,6,7,8] have been developed, in order to study the fluid flow in gas and induction stirred ladles. These models were used to gain more insight in the industrial processes used in ladle treatment of steel. In this paper a computational fluid dynamics model of a 20 Kg laboratory scaled induction ladle (Situated at the Dept. of Material Science and Engineering - KTH) is presented. This particular laboratory furnace can be equipped with an electromagnetic stirrer, which can be used to agitate the steel melt. The CFD model so developed will make it feasible to have information about the fluid flow in this particular laboratory furnace. This information would promote the analysis of experimental results and the implementation of new strategies. The objective of this paper is to obtain an understanding and insight of the ladle refining process by solving the electromagnetic force field and predict the flow pattern produced by these force fields using a single straight induction stirrer, with the help of 20Kg laboratory scaled furnace. The size of this 20Kg laboratory scaled furnace is very small compared to the size of electromagnetic stirrer because of which the magnetic field inside the ladle will essentially be two dimensional. The magnetic field component in third dimension (Y-direction) is very small compared to other two (X and Z direction) hence a two dimensional model also provides a better understating of the model. The flow field produced by a straight induction stirrer is of complex nature due to the three dimensional electromagnetic force and the flow phenomenon. In order to provide more information about the stirring of molten steel within the ladle, which is essentially a three-dimensional phenomenon, a three-dimensional model is also presented incorporating equations governing the fluid flow as well as the electromagnetic forces in the system. Both the models two and three – dimensional are developed in two parts. First, the calculation of electromagnetic forces, which is done with the help of FEMLAB3.1 and second, using these electromagnetic forces (Lorenz forces) as the source term for solving incompressible Navier stokes equation to compute the velocity profile of the agitated melt, which is done by combining the Electromagnetic module with Chemical Engineering module of commercial software FEMLAB 3.1. The CFD model so developed is verified using experimental measurements of the magnetic flux in the laboratory furnace. This CFD model of the induction ladle is developed with the possible extension of the model in mind, i.e. it should be easy to expand the model to incorporate temperature as well as transport of chemical species and non – metallic inclusions.

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1 Introduction

The CFD model of the induction stirred ladle consists of two main parts. First, the electromagnetic Lorenz forces (which are included in the steel melt when the stirrer is on) are calculated. Secondly, the resulting flow field caused by the Lorenz forces is calculated. The physical orientation of the induction stirrer and the ladle is shown in figure 1. The induction coils in-side the stirrer produces a high frequency magnetic field. This magnetic field induces currents on the surface of the conductor whose distribution is such as to shield the interior of the conductor from the imposed field. These currents are restricted to a thin surface layer of thickness, called as skin depth [9]. In fact, we take as our definition of the term ‘high frequency’ that the skin depth thickness is much less than any relevant geometric length scale, say the characteristic size of the body. Now the fact that magnetic field is shielded from the interior of the conductor is not, in itself, particularly important in metallurgical magnetohydrodynamics (MHD).

However, the existence of thin surface layer of induced current is useful. Opposite current repel each other, and so the conductor will experience a sidewise repulsion force. Moreover, the induced current will heat the conductor. It is the ability of high frequency conductor to repel and heat conducting material, liquid or solid [9]. The above literature can now be explained with the help of following mathematical formulations.

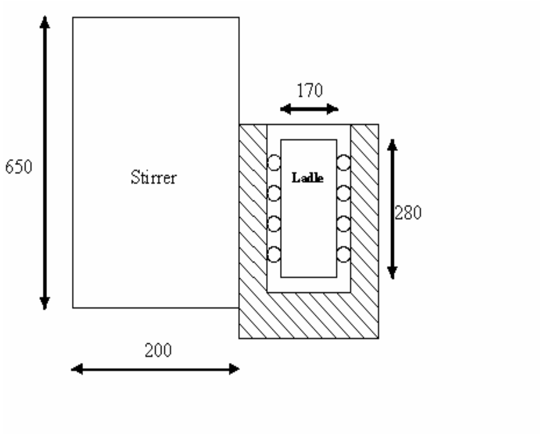

Fig. 1 Induction stirrer and the ladle, all dimensions in *mm*

2 Governing equation for calculating electromagnetic forces

B – Magnetic Flux density.
J – Current density.
E – Electric potential.
H – Magnetic field density.
μ – Magnetic permeability.
σ – Electrical conductivity.
U – Velocity of the conductor in the magnetic field.
F – Volumetric Lorenz force.

$$\nabla \times B = J \tag{1}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2}$$

$$B = \mu H \tag{3}$$

and ohm's law

$$J = \sigma(E + (U \times B)) \tag{4}$$

The Lorenz force due to interaction of the electromagnetic fields is given by

$$F = J \times B \tag{5}$$

To determine the electromagnetic force, it must first be decided whether to solve J or B in the conducting domain, because J and B are linked to each other by the equation 1 and 4.

The above equations can now be manipulated to form a differential equation in B which is given as:

$$\frac{\partial B}{\partial t} = \frac{1}{\sigma\mu}\nabla^2 B + \nabla \times U \times B \tag{6}$$

Since the magnetic Reynolds number is low in this metallurgical process, the second term on the right of the above equation can be neglected [3]. Thus the above equation reduces to:

$$\frac{\partial B}{\partial t} = \frac{1}{\sigma\mu}\nabla^2 B \tag{7}$$

This is a magnetic diffusion equation. In steady state the equation reduced to an elliptic partial differential equation. The above magnetic diffusion equation is now solved with the appropriate boundary conditions to obtain the values of B_x, B_y and B_z consecutively with the help of the magnetic fields the induced current density can be calculated which is given by equation 1.

Since the electromagnetic field is varying sinusoidally with time, it is necessary to represent the Lorenz force in equation 5 in the time averaged form. The components of the Lorenz forces in x, y and z directions are given as:

$$F_x = J_y * B_z - J_z * B_y \tag{8}$$

$$F_y = J_z * B_x - J_x * B_z \tag{9}$$

$$F_z = J_x * B_y - J_y * B_x \tag{10}$$

In order to compute the velocity profile in the liquid metal bath the Lorenz forces calculated above are then used as the source term to solve the incompressible Navier-Stokes equation.

3 Equation governing flow field calculations

This paper contains a two-dimensional as well as a three-dimensional model of the inductively stirred ladle. Therefore, all the equations are presented in a three-dimensional Cartesian coordinate system as it is quite easy to obtain equation for two-dimensional model from the equation governing three-dimensional phenomenon. The equation of motion for the liquid metal in the ladle is given in Cartesian coordinate system as follows:

u – Velocity along x-direction.
v – Velocity along y-direction.
w –Velocity along z-direction.
P – Pressure.
ρ – Density of the fluid.
η – Viscosity of the fluid.

Fx, Fy and Fz are the volumetric Lorenz forces along x, y and z direction.
t – Time.

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad 11$$

Momentum equation

x component

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})+F_x \qquad 12$$

y component

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=-\frac{\partial p}{\partial y}+\eta(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2})+F_y \qquad 13$$

z component

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})+F_z \qquad 14$$

Turbulence modeling

The *k*- ε is one of the most used turbulence models for industrial applications. The *k*- ε model belongs to the class of turbulence models called two-equation models. In this model, two extra transport equations are solved for the turbulence kinetic energy, *k*, and the dissipation rate of turbulence energy, ε. Further details about the equation governing *k*- ε turbulence incorporated in FEMLAB 3.1 can be found in [10].

4 Modeling of Induction stirred ladle in FEMLAB 3.1

In this section CFD-Model of induction stirred ladle is presented in both two and three-dimension. First a two-dimensional model is presented followed by a three-dimensional model.

4.1 Two-Dimensional model.

The induction coils were modeled as two separate conducting domains and the ladle was modeled as another conducting domain. The coil and the ladle domains were enclosed in another domain where the boundary conditions were set as magnetic insulation at the boundary of enclosing domain. In FEMLAB 3.1 the model is generated by coupling of electromagnetic quasi static with incompressible Navier-Stokes module with *k-ε* turbulence modeling. A more detailed description can be found in [8]. The dimensions of the ladle and the stirrer are shown in figure 1.The straight stirrer with a single phase induction coil (which can easily be converted to 2 or 3 phase induction coil) was used in this model. Single phase current of 1000 ampere at frequency of 50 Hz was used. The table 1 gives the physical and electrical properties used in the model.
The volumetric Lorenz forces in X and Y directions are given as

$$F_x = real(Jiz_{qa} * conj(By_{qa})) \qquad 15$$

$$F_y = real(Jiz_{qa} * conj(Bx_{qa})) \qquad 16$$

Where Jiz_{qa} , By_{qa} and Bx_{qa} are the induced current densities and magnetic flux densities respectively. For further details please see the FEMLAB 3.1 code included in [8].

Physical and electrical property	Values
ρ - Density of Molten steel used is	8.586e3-0.8567*T $Kg.m^{-3}$
T - Temperature of the molten steel	1873 K
ν - Kinematic Viscosity of Molten steel	((0.3147e-3)*exp (46480/ (8.3144*T)))/ (8.586e3-0.856*T) $m^2.s^{-1}$
A - Area of cross section of coils	0.65*0.005 m^2
I_o - Current in the coil	1000 Ampere
J - Current density in the coils	$I_o / Area$
σ - Conductivity of Molten steel in ladle	1.04e6 $S.m^{-1}$

Table 1. The **physical** and electrical properties in two-dimensional case.

4.2 Boundary condition for two-dimensional model.

For the electromagnetic quasi static module in FEMLAB 3.1 all the three domains were set to active and the boundary conditions were set to magnetic insulation at boundary of the domain enclosing the coil and the ladle. For the *k*-ε turbulence module only ladle domain was kept active and the boundary conditions were set to no-slip at the boundary of the ladle. A_z = 0 i.e. magnetic potential was set to zero at the boundary corresponding to magnetic insulation. u = 0, k = 0 and $\partial\varepsilon/\partial n = 0$; Boundary conditions corresponding to no-slip.

4.3 Mesh setting for two-dimensional model.

The initialized mesh consists of 7676 mesh elements and 18715 of degrees of freedom. Then a mesh refinement was applied on the ladle in order to get better resolution of the forces and the velocity inside the ladle. The figure 2 shows the mesh generated on the whole domain. From this figure it can be seen that mesh is made denser close to the ladle and the stirrer, and is kept coarser at the boundary.

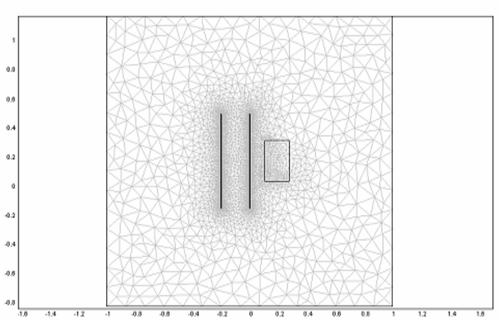

Fig 2. Initialized mesh and enclosing domain.

4.4 Solver setting for two-dimensional model.

First the solution was obtained for the electromagnetic quasi static module with the use of time-harmonic, direct linear solver. In this way the force fields were calculated and these force fields were used as source terms to solve incompressible Navier-stokes equation with *k*-ε turbulence modeling. Where time-harmonic, non-linear iterative solver was used.

4.5 Three-dimensional model.

In the three dimensional model of the induction stirred ladle furnace in FEMLAB 3.1, the stirrer was modeled as a coil for which a Matlab code was first generated and then transported to FEMLAB 3.1. The current was then flown into the coil as line segment. The ladle was modeled as a conducting cylindrical domain. The figure 3 gives a better picture of how the three-dimensional system was modeled in FEMLAB 3.1.

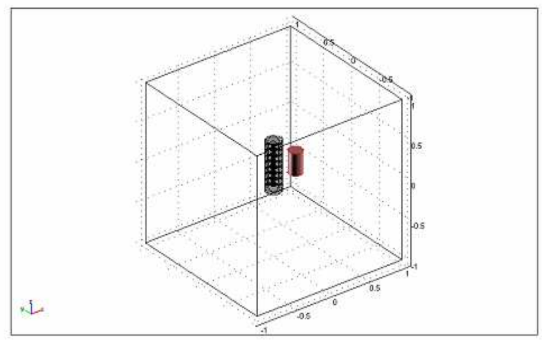

Fig 3 Arrangement of ladle and stirred in three- dimensional model.

4.6 Three-dimensional model settings in FEMLAB3.1.

In three-dimensional model the coil and the ladle domains were enclosed in another domain with the boundary conditions set as magnetic insulation at the boundary of the enclosing domain. The three-dimensional model was generated similar to the two-dimensional model by coupling of Electromagnetic quasi static and Incompressible Navier-Stokes module with *k-ε* turbulence modeling. The dimensions of the ladle are same as in the two-dimensional case.

The straight stirrer used for the above computations was modeled using single phase induction coil (which can easily be converted to 2 or 3 phase induction coil). The specifications of the coil are as follows:

Height of the coil = 0.65 m
No of winding used in modeling = 41
Diameter of the coil = 0.20 m

Single phase current of 406 ampere at frequency of 50 Hz was used. Table 2 shows the physical and electrical properties of the coil used in three-dimensional model and the induced Lorenz forces are given as:

$$F_x = real(Jiy_{aqv} * conj(Bz_{qav}) - Jiz_{qav} * conj(By_{qav})) \quad 17$$

$$F_y = real(Jiz_{aqv} * conj(Bx_{qav}) - Jix_{qav} * conj(Bz_{qav})) \quad 18$$

$$F_z = real(Jiz_{aqv} * conj(Bx_{qav}) - Jix_{qav} * conj(Bz_{qav})) \quad 19$$

Where $Jiz_{qav}, Jix_{qav}, Jiy_{qav}, By_{qav}, Bz_{qav}$ and Bix_{qav} are the induced current densities and magnetic flux densities respectively.

Physical and electrical property	Values
ρ - Density of Molten steel used is	7850 $Kg.m^{-3}$
T - Temperature of the Molten steel	1873 K
ν - Kinematic Viscosity of Molten steel	1 $m^2.s^{-1}$ (To achieve faster convergence without turbulence)
I_o - Current in the coil	406 Ampere
σ - Conductivity of Molten steel in ladle	1.04e6 $S.m^{-1}$

Table 2. Physical and electrical Properties in three-dimensional case

4.7 Boundary conditions for the three-dimensional model.

For the electromagnetic quasi static module all the three domains were kept active and the boundary condition were set to magnetic insulation (*Axn* = 0) at boundary of the domain enclosing the coil and the ladle. In the steady state incompressible Navier-stokes module only ladle domain was kept active and the boundary conditions were set to *no-slip (u=0)* at the boundary of the ladle. Pressure was defined as a point constraint by choosing a point inside the ladle and setting the pressure value for that point to one.

4.8 Mesh setting for the Three-dimensional model.

The initialized mesh consists of 24807 mesh elements and 34178 degree of freedom. Then a mesh refinement was applied over the domain in order to get better resolution of the forces and the velocity in the ladle. The figure 4 shows the initialized mesh over the whole domain.

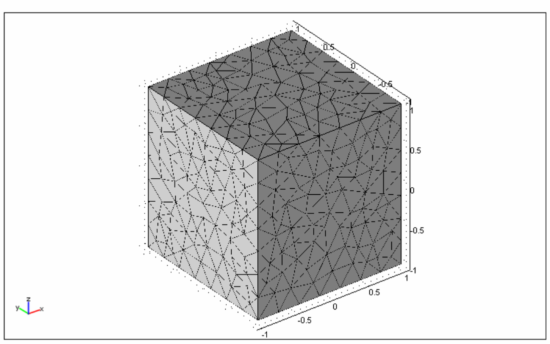

Fig 4. Initialized mesh on the domain in three-dimensional model.

4.9 Solver settings for the Three-dimensional model.

First the solution was obtained for the electromagnetic quasi static module with the use of time-harmonic, direct linear solver. In this way the force fields were calculated and these force fields were used in the steady state incompressible Navier-stokes module. Where time-harmonic, non-linear iterative solver was used.

5 Results

In this section results for the model of inductively stirred ladle are presented. First, the results obtained

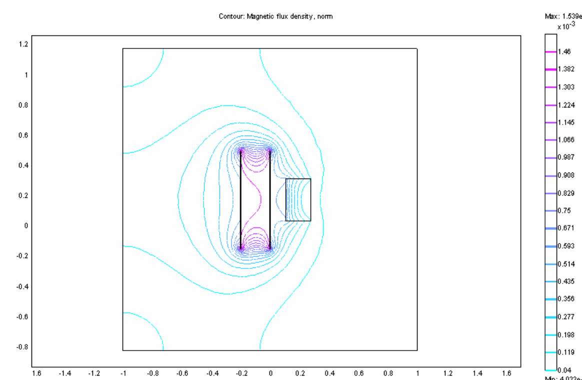

Fig 5. Contours of Magnetic Flux density.

for the two-dimensional model are presented followed by the results for the three- dimensional model. Also, a comparison is made between the experimentally computed results of the magnetic flux with that of the magnetic flux values obtained by the model developed using FEMLAB 3.0. Also, an analysis was done how the velocity field changes with the orientation of ladle with respect to the inductor, both in two and three-dimensional.

5.1 Results for two -dimensional model of induction stirred ladle.

The figure 5 shows the contour plot of the magnetic flux density, from the plot its is very clear that the magnetic flux density is high near the surface of the stirrer and decreases gradually on moving away from the stirrer. It can also be seen that the surface of the ladle near to the stirrer has more induced magnetic density compared to the surface away from the stirrer.

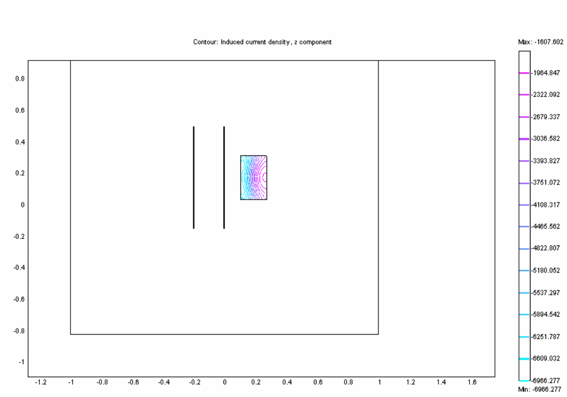

Fig 6. Induced current density *Jiz.*

The figure 6 shows the contour plot of the induced current density in the ladle, induced current density is varying inside the ladle. The surface of the ladle which is very near to the stirrer has a higher induced current density compared to the surface away from the stirrer. The reason being, when an oscillating magnetic field is applied near to the surface of an electrically conducting medium, the magnetic field will penetrate to a finite distance into the medium. This will result in generation of induced current near the surface of the conducting medium which will oppose the applied magnetic field [9].

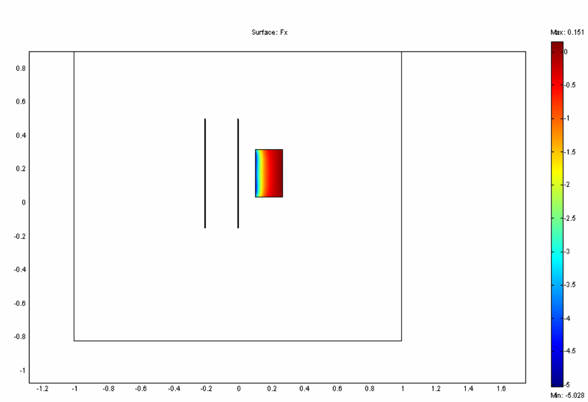

Fig 7. The volumetric induced force Fx in the ladle.

From the figure 7 showing the surface plot it can be seen how the induced Lorenz force is varying inside the ladle domain. The plots of the magnetic flux and the induced current have shown that the induced magnetic field and the induced current is relatively higher inside the ladle domain near to the stirrer. The Lorenz force which is given as cross product of

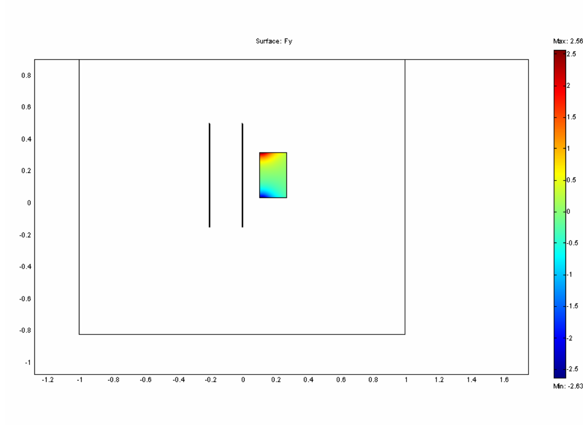

Fig 8. The volumetric induce force Fy inside the ladle.

induced current density and the induced magnetic field is more near to the surface. The figure 8 shows the surface plot of the induced force. The induced Lorenz force at the bottom left corner of the ladle is in opposite direction to the induced force at the top left corner of the ladle. This also gives an explanation for circular motion of the molten steel inside the ladle.

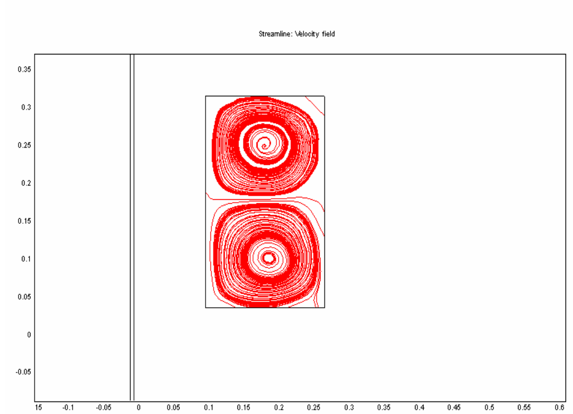

Fig 9. Velocity profile of the fluid flow inside the ladle.

The streamline plot in figure 9 of the induced velocity inside the ladle shows the flow pattern of the molten steel. The flow profile is quite similar to the flow profile speculated in the experimental measurement carried out by Moore [11] for single phase induction stirrer. The contour plot of the velocity in figure 10 shows how the velocity values are changing inside the ladle. The velocity is almost zero at the surface of the ladle, because of the no-slip boundary condition imposed at the sides of the ladle, than, the velocity is higher farther away from the surface.

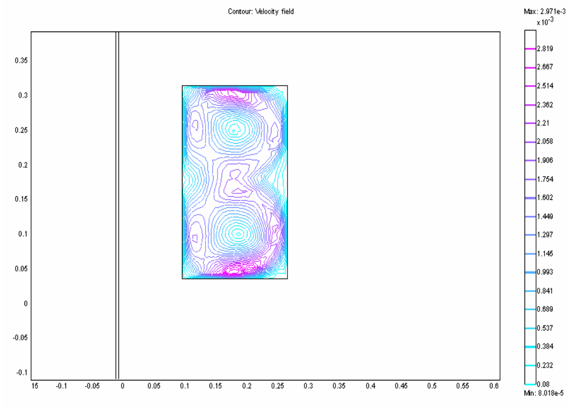

Fig 10. Contour plot of the velocity inside the ladle.

5.2 Results for three -dimensional model of induction stirred ladle.

In this section results for the three dimensional model of the induction stirred ladle are presented. The solution was first obtained for the electromagnetic part and then these results were used as input for the fluid flow part of the problem.

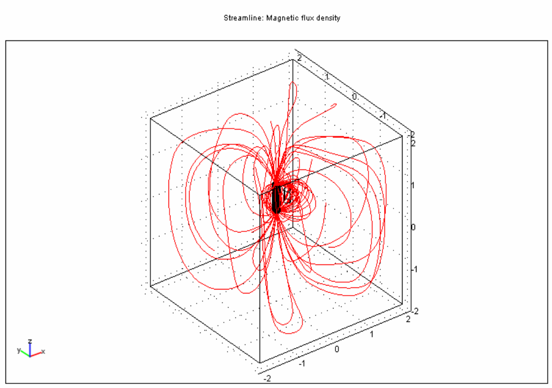

Fig 11. Magnetic flux in the form of magnetic field lines.

The figure 11 shows the streamline plot of the magnetic flux density, the magnetic field diffuses on moving away from the stirrer. The streamline density indicates that the magnetic flux if very high at the centre of the coil/stirrer and reduces exponentially on moving away from the stirrer radially. Figure 12 shows the cross-sectional plot of the magnetic flux density inside the ladle and figure 13 shows the plot of induced currents generated inside the ladle.

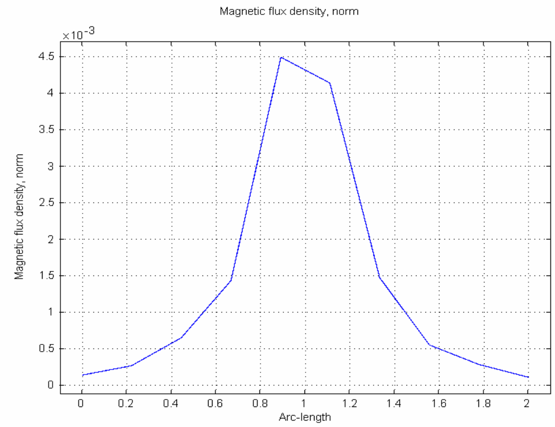

Fig12. Cross-section of the magnetic flux density inside the ladle.

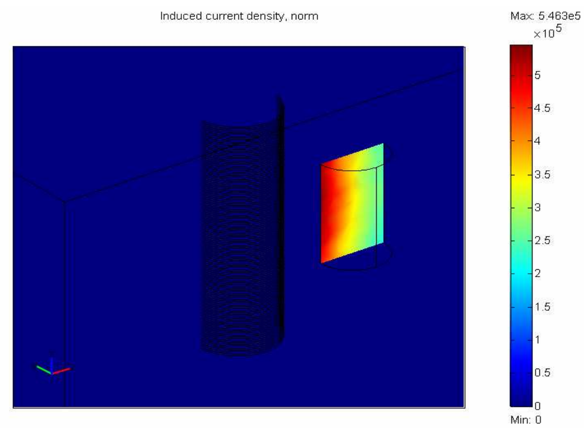

Fig 13. Induced current density inside the ladle

It can be seen from the figures that the induced current density is more at the face of the ladle near to the induction stirrer as observed in two-dimensional model.

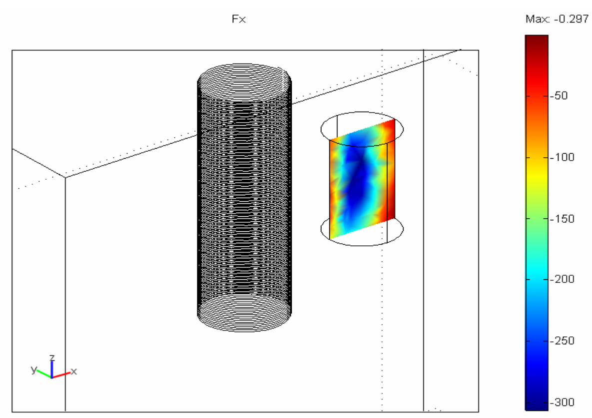

Fig 14. Volumetric Lorenz force Fx inside the ladle.

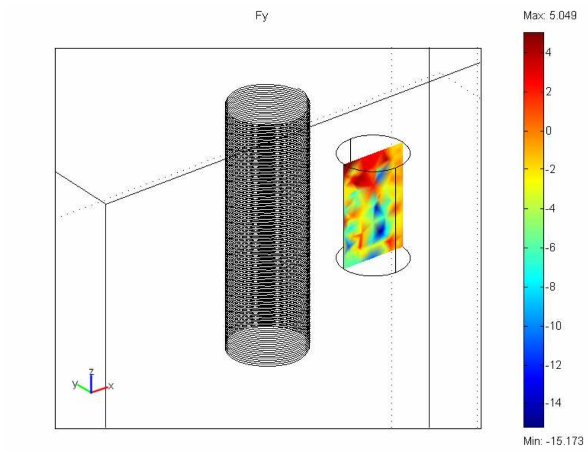

Fig 15. Volumetric Lorenz force Fy inside the ladle.

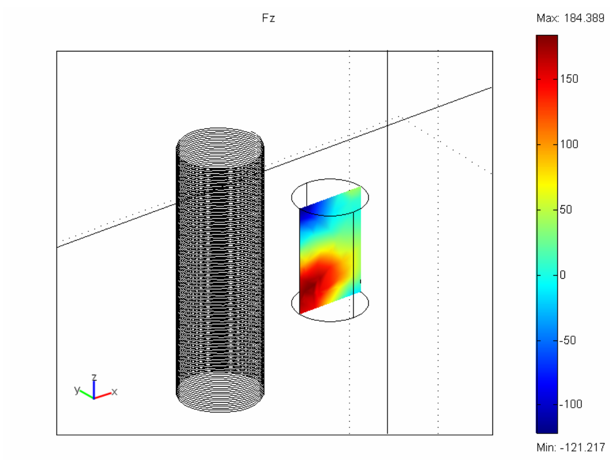

Fig 16. Volumetric Lorenz force Fz inside the ladle.

The figure 14, 15 and 16 shows the surface plot of the Lorenz force inside the ladle. In figure 16 the induced force is in different direction at the bottom left and the top right position inside the ladle similar to the observation made in two-dimensional model, hence resulting in circular rotation of molten steel inside the ladle. Figure 17 shows the velocity profile of the induced flow inside the ladle.

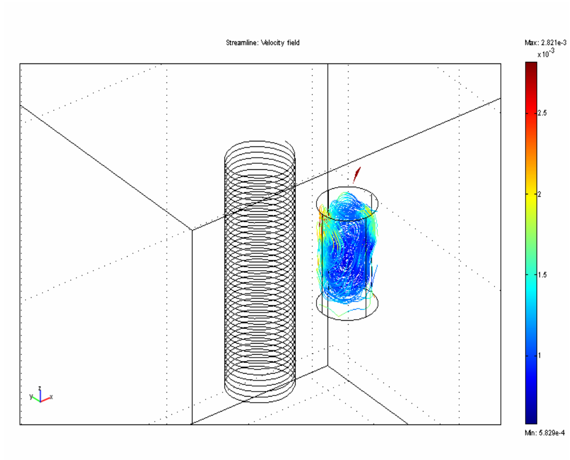

Fig 17. Velocity profile of the fluid flow inside the ladle.

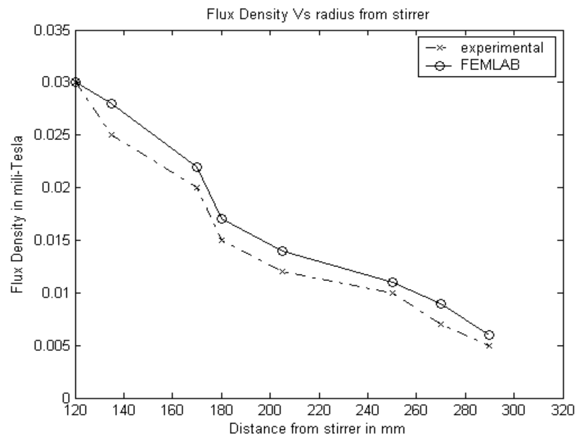

Fig 18. Comparison of the magnetic flux obtained using FEMLAB 3.1 with experimental results.

6 Summary

It was observed that as the ladle is brought closer to the stirrer the induced current density inside the melt increases thereby increasing the induced force field and hence the velocity of the melt increases. These results obtained from modeling of the ladle refining process with the help of FEMLAB 3.1, confirm the physics involved in this phenomenon. These results also match with the velocity profile of the melt calculated by Moore [11] using experimental setup for single phase induction stirrer. In order to confirm that the results obtained by this FEMLAB 3.1 generated model of the inductively stirred ladle a comparison was made between the magnetic flux values computed experimentally form the laboratory scaled ladle with magnetic flux values computed by the FEMLAB 3.1 generated model. The comparison is shown in figure 18. The experimental measurements were carried out by ABB for the laboratory scaled furnace for the different orientation of the stirrer with respect to the ladle. From the comparison it can seen that the flux values calculated using the FEMLAB 3.1 model are in good agreement with the experimental measurements carried out by ABB. This also validates the FEMLAB 3.1 model.

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